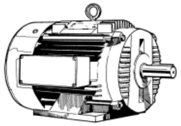
LECTURE NOTES

ELECTRICAL MACHINES

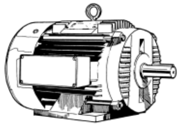
EE352

FALL 20103

Dr : MUSTAFA AL-REFAI

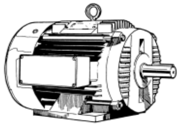


LECTURE ONE



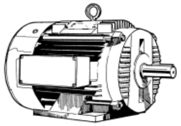
Tests

- ❑ **Three (3) tests are scheduled. Each test has theoretical questions and problems.**
  - **The dates of the tests are:**
  - **FIRST TEST WILL BE HELED DURING FIRST LECTURE ON THE FIRST EXAM WEEK**
  - **SECOND TEST WILL BE HELED DURING FIRST LECTURE ON THE SECOND EXAM WEEK**
- ❑ **Final exam**



AIMS OF THIS COURSE

- The objective of this course is to study the devices used in the interconversion of electric and mechanical energy, with emphasis placed on electromagnetic rotating machinery.
- The transformer, although not an electromechanical-energy-conversion device, is an important component of the overall energy-conversion process.
- Practically all transformers and electric machinery use ferro-magnetic material for shaping and directing the magnetic fields that acts as the medium for transferring and converting energy. Permanent-magnet materials are also widely used.
- The ability to analyze and describe systems containing magnetic materials is essential for designing and understanding electromechanical-energy-conversion devices.
- The techniques of magnetic-circuit analysis, which represent algebraic approximations to exact field-theory solutions, are widely used in the study of electromechanical-energy-conversion devices.



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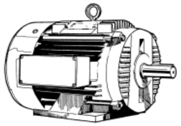
Chapter 2 Transformers

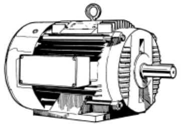
Chapter 3 Electromechanical-Energy-Conversion Principles

Chapter 4 Introduction to Rotating Machines



Units

- system of units
  - System International (SI)
  - English
- conversion of units
  - scale conversions
  - system of units conversions
- the per-unit system of measurements



System of Units

SI common units

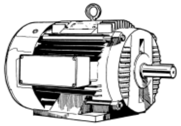
– some used in this course are shown in this table

– three general areas we will be using:

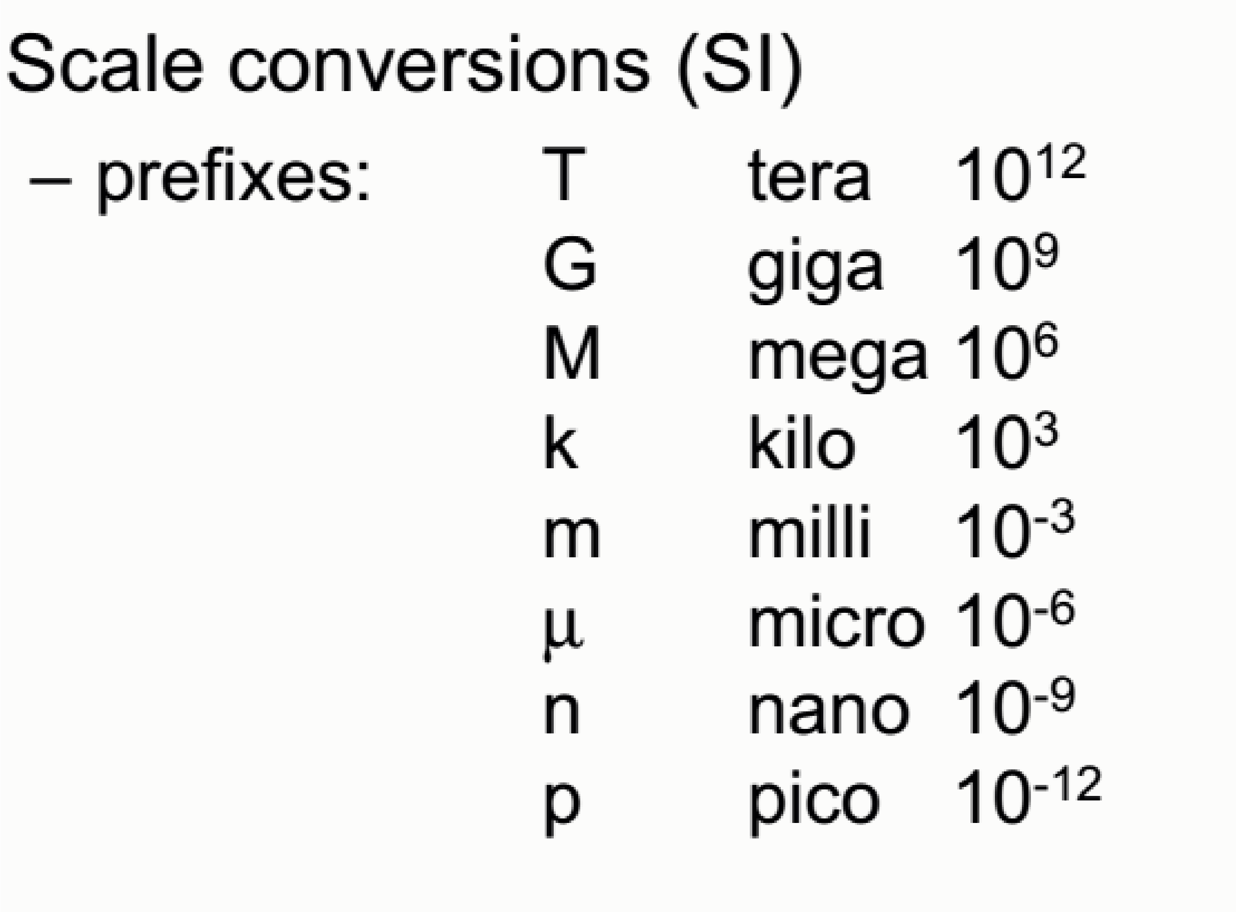
- electrical
- mechanical
- thermal

– working in SI units are often easier than in English unit

Quantity	SI unit	Symbol
Mechanical		
Angle	radian	rad
Energy	joule	J
Force	Newton	N
Mass	kilogram	kg
Power	watt	W
Angular velocity	radians per second	rad/s
Thermal		
Heat	Joule	J
Power	watt	W
Specific Heat	joule per (kg K)	J/(kg K)
Temperature	kelvin	K
Thermal conductivity	watt per (meter K)	W/(m K)
Electrical and Magnetic		
Energy	joule	J
Frequency	hertz	Hz
Inductance	henry	H
Power	watt	W
Magnetic field	ampere per meter	A/m
Magnetic flux	weber	Wb
Magnetic flux density	tesla	T
Magneto-motive forc	ampere	A



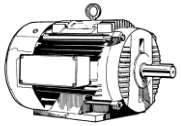
Conversion of Units



Scale conversions (SI)

– prefixes:

T	tera	10^{12}
G	giga	10^{9}
M	mega	10^{6}
k	kilo	10^{3}
m	milli	10^{-3}
μ	micro	10^{-6}
n	nano	10^{-9}
p	pico	10^{-12}



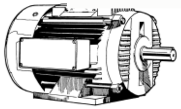
Conversion of Units

- System of units conversions
  - check for consistency
    - stay within a given quantity, e.g. impedance or power
    - avoid crossing quantities
  - always show conversion steps in your work
  - example

convert 850 Btu into kilowatt-hours

$$850\,\mathrm{Btu}\cdot\frac{1.055\,\mathrm{kJ}}{\mathrm{Btu}}\cdot\frac{\mathrm{h}}{3600\,\mathrm{s}}\cdot\frac{\mathrm{W\cdot s}}{\mathrm{J}}=0.25\,\mathrm{kW\cdot h}$$

850 Btu = 0.25 kWh

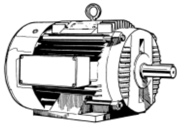


Per Unit Measurement System

- A measurement system for giving a better idea of the size of something
  - similar to the numbers used in baseball statistics
  - comparable to how we use the gas gage on vehicles
- A per unit measurement always involves a base value
  - this conversion unit is called the *per unit base*
  - the converted value is expressed in terms of *per unit*
  - example

    convert 115 kW to a per unit value on a 1 MVA base

$$\frac{115\text{kW}}{1\text{MVA}}=0.115\ per\ \ unit$$



Per Unit Bases

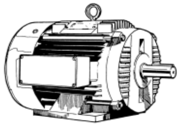
- Base values for different engineering quantities must be consistent once two or more bases have been defined
  - example
    - show the current base and impedance base for a system that has a power base of 1 kVA and a voltage base of 240 V

$$I_{base} = \frac{S_{base}}{V_{base}} = \frac{1\text{kVA}}{240\text{V}} = 4.17\text{A} \qquad Z_{base} = \frac{V_{base}}{I_{base}} = \frac{240\text{V}}{4.17\text{A}} = 57.6\Omega$$

    - what are the engineering values for a device operating at 0.98 per unit voltage with a load of j0.7 per unit impedance

$$V = V_{base} \cdot V_{pu} = 240\text{V}(0.98) = 235.2\text{V}$$

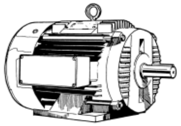
$$Z = Z_{base} \cdot Z_{pu} = 57.6\Omega(j0.7) = j40.32\Omega$$



Magnetism

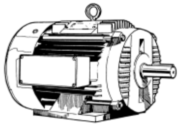
and

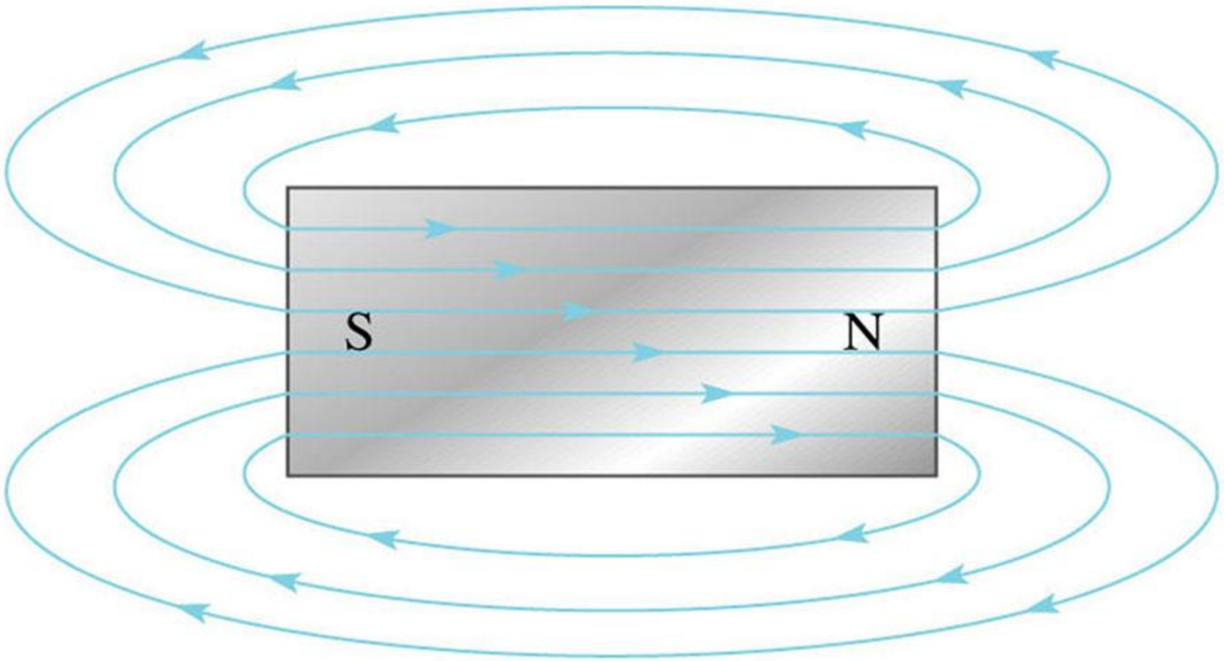
Electromagnetism



The Magnetic Field

- A permanent magnet has a magnetic field surrounding it.
- Consists of *lines of force* that radiate from the north pole to the south pole and back to the north pole through the magnetic material.



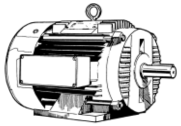


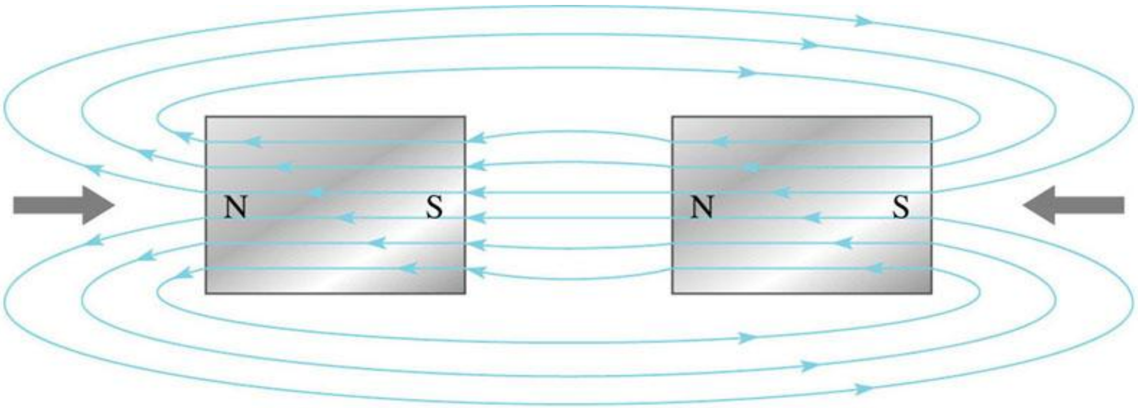
Blue lines represent only a few of the many magnetic lines of force in the magnetic field.

Figure 1 Magnetic lines of force around a bar magnet.

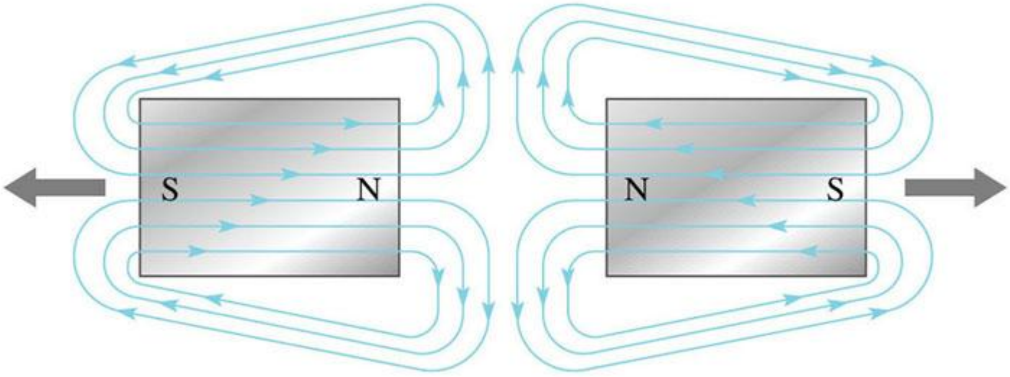
The Magnetic Field

- Consists of lines of force, (or flux lines), that radiate from the north pole (N) to the south pole (S) and back to the N. pole through the magnetic material.
- The many lines surround the magnet in 3 dimensions.
- Lines shrink to the smallest possible size and blend together;- although they do not touch.
- Forms a continuous magnetic field surrounding the magnet.



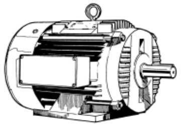


(a) Unlike poles attract.



(b) Like poles repel.

Figure 2 Magnetic attraction and repulsion



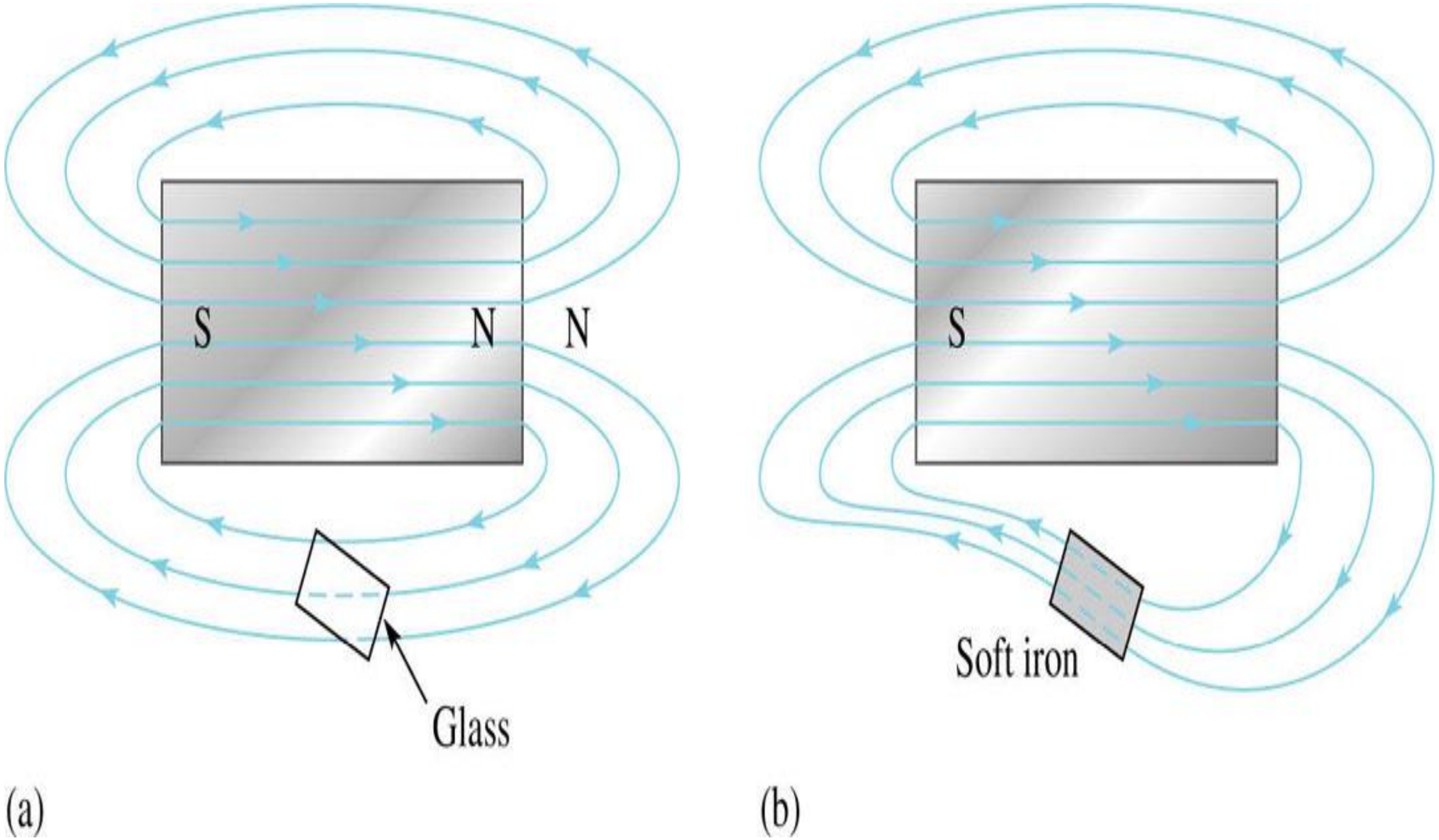
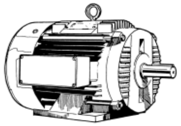


Fig. 3 Effect of (a) nonmagnetic and (b) magnetic materials on a magnetic field.

Magnetic Flux, Φ

- The group of force lines going from the N. pole to the S. pole of a magnet is called the magnetic flux, symbolized by Φ (phi).
- No. of lines of force in a magnetic field determines the value of the flux.
- The more lines of force, the greater the flux and the stronger the magnetic field.
- Unit of magnetic flux is the weber (Wb)
- One weber = 10^8 lines.

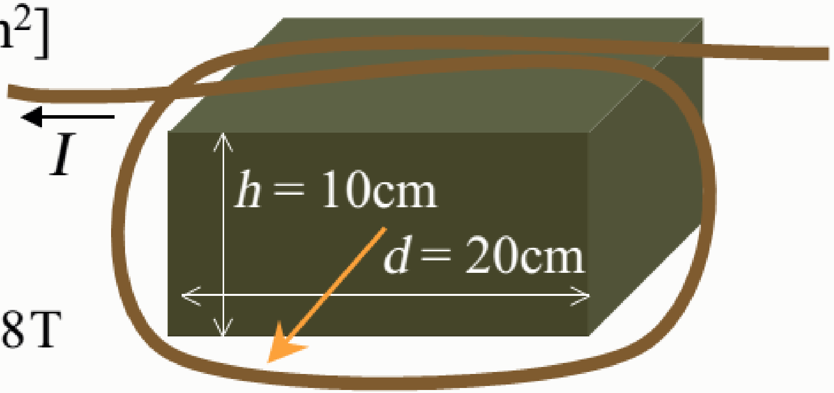


Magnetic Flux Density

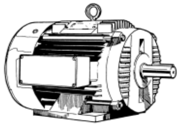
- For a magnetic flux ϕ, there exists a **magnetic flux density**, B, given by $B=\dfrac{\phi}{A}$ where
  - B = flux density [T] (tesla)
  - ϕ = flux in a component [Wb] (weber)
  - A = cross section area [m^2]
  - example

    find the flux density

$$B=\frac{\phi}{d\,h}=\frac{360\times10^{-6}\ \text{Wb}}{(0.01\,\text{m})(0.02\,\text{m})}=1.8\text{T}$$



$\phi = 360\mu$Wb

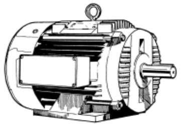


Magnetic Flux Density, (B)

- Is the amount of flux per unit area perpendicular to the magnetic field.
- Its symbol is B, and its unit is the tesla (T).
- One tesla = one weber/square meter (Wb/m^2).
- The following expresses the flux density:

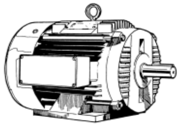
$$\mathbf{B = \frac{\Phi}{A}}$$

- Φ is the flux, A is the c.s.a in square meters (m^2) of the magnetic field.



The Gauss

- The tesla (T) is the SI unit for flux density, another unit called the gauss, from the CGS (centimeter-gram-second) system, is sometimes used (10^4 gauss = 1T).
- The instrument used to measure flux density is the gaussmeter.



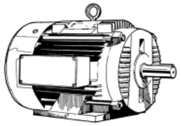
Example 1

- Find the flux density in a magnetic field in which the flux in $0.1m^2$ is 800μWb.
- Solution

$$B = \Phi/A$$

$$= 800\mu Wb/0.1m^2$$

$$= 8000\mu T$$



Example 2

- A magnetic pole face has a rectangular section having dimensions 200 mm by 100 mm. If the total flux emerging from the pole is 150 μ Wb. Calculate the flux density.

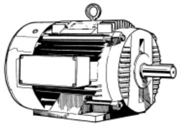
Solution:

- $\Phi = 150\ \mu\text{ Wb} = 150 \times 10^{-6}$ Wb
- c.s.a = $200 \times 100 = 20000\text{ mm}^2 = 20000 \times 10^{-6}\text{ m}^2$

Flux Density, $B = \Phi/A$

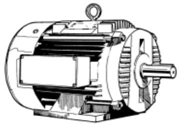
$= 150 \times 10^{-6}/20000 \times 10^{-6}$

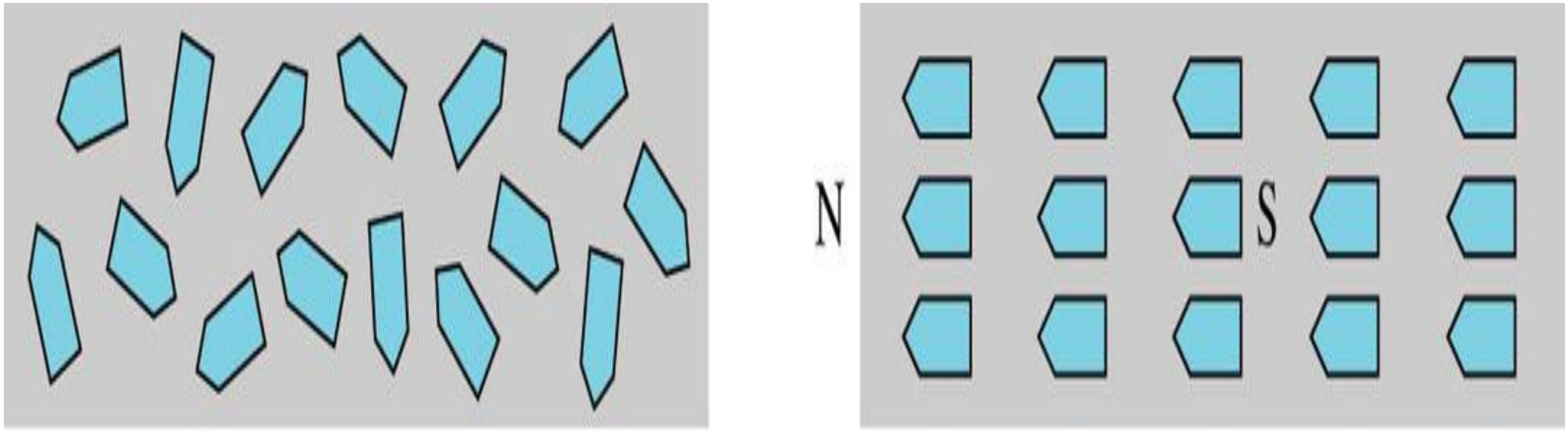
$= 0.0075$ T or 7.5mT



How Materials Become Magnetised

- Ferromagnetic materials become magnetised when placed in the magnetic field of a magnet.
- We have all seen a permanent magnet pick up paper clips, nails, or iron filings.
- Objects becomes magnetised under the influence of the permanent magnetic field and becomes attracted to the magnet.
- When removed from the magnetic field, object tends to lose its magnetism.
- Ferromagnetic materials have minute magnetic domains created within their atomic structure by the orbital motion and spin of electrons.
- These domains can be viewed as very small bar magnets with N. and S. poles.

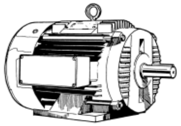




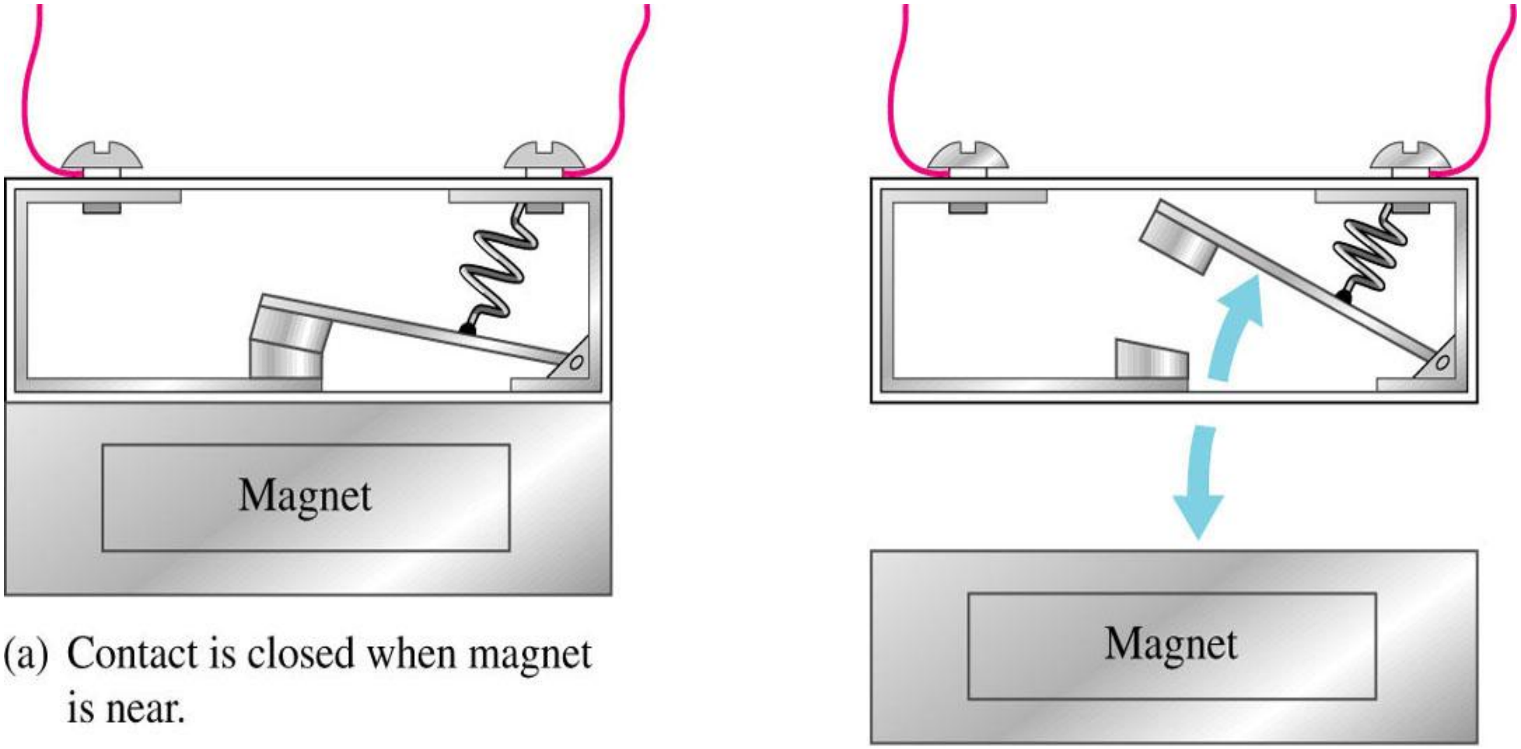
(a) The magnetic domains (N ⬠ S) are randomly oriented in the unmagnetized material.

(b) The magnetic domains become aligned when the material is magnetized.

Figure 4 Magnetic domains in (a) an unmagnetized and in (b) a magnetised material.



Application Example



(a) Contact is closed when magnet is near.

(b) Contact opens when magnet is moved away.

Figure 5 Operation of a magnetic switch

Quiz 1

- When the North poles of two magnets are placed close together, do they repel or attract each other?

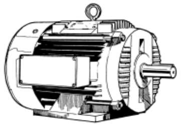
Ans: The North Poles repel

- **What is magnetic flux?**

Ans: Magnetic flux is the group of lines of force that make up a magnetic field.

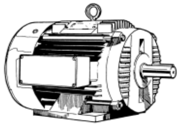
- What is the flux density when $\Phi = 4.5\mu Wb$ and $A = 5 \times 10^{-3}\ m^2$?

Ans: $B = \Phi/A = 900\mu T$



Electromagnetism

- Is the production of a magnetic field by current in a conductor.
- Many types of useful devices such as tape recorders, electric motors, speakers, solenoids, and relays are based on electromagnetism.



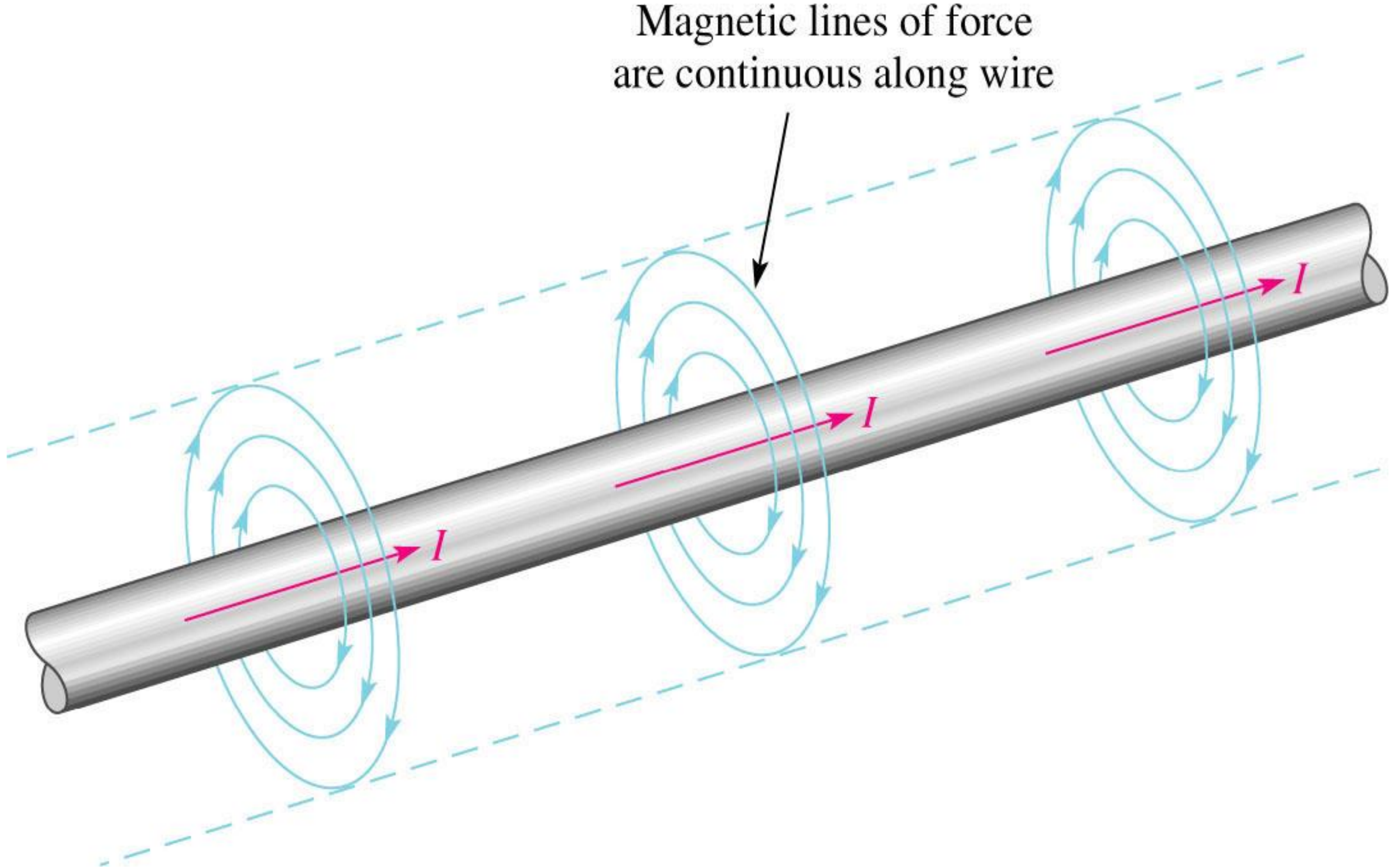
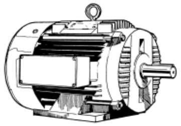

Magnetic lines of force are continuous along wire
I
I
I

Fig. 6 Magnetic field around a current-carrying conductor



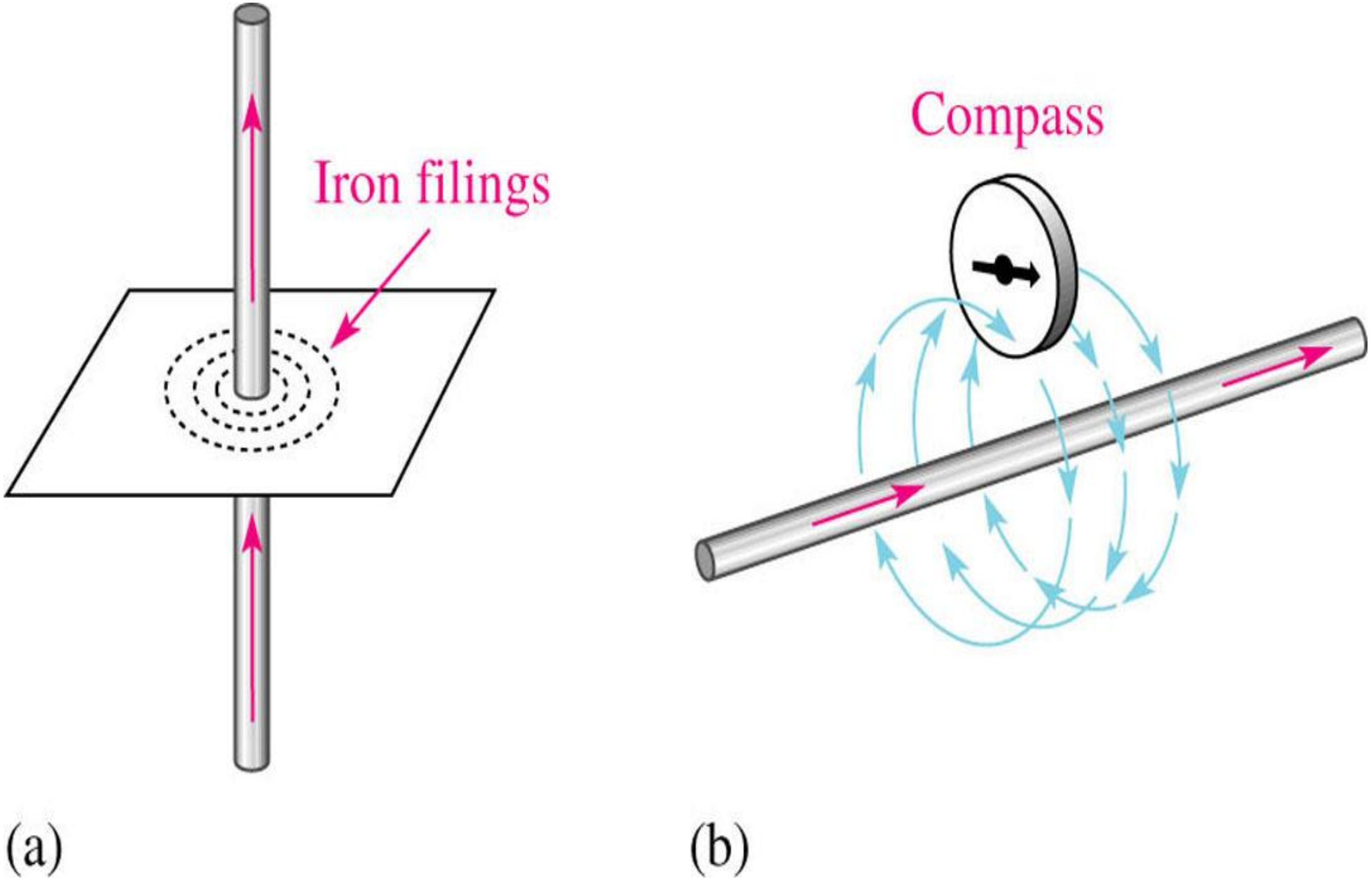


Figure 7 Visible effects of an electromagnetic field

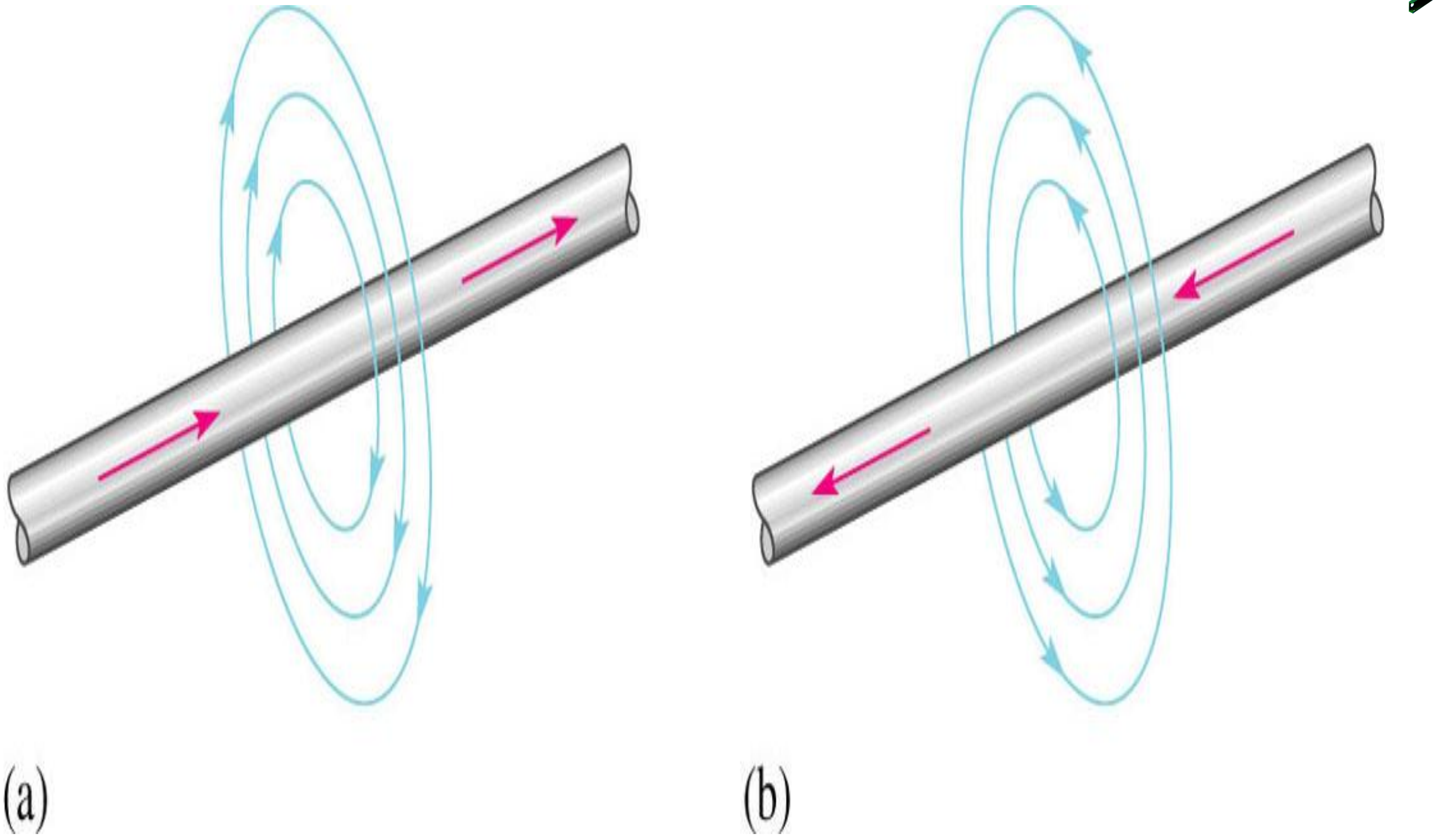


Fig. 8 Magnetic lines of force around a current-carrying conductor

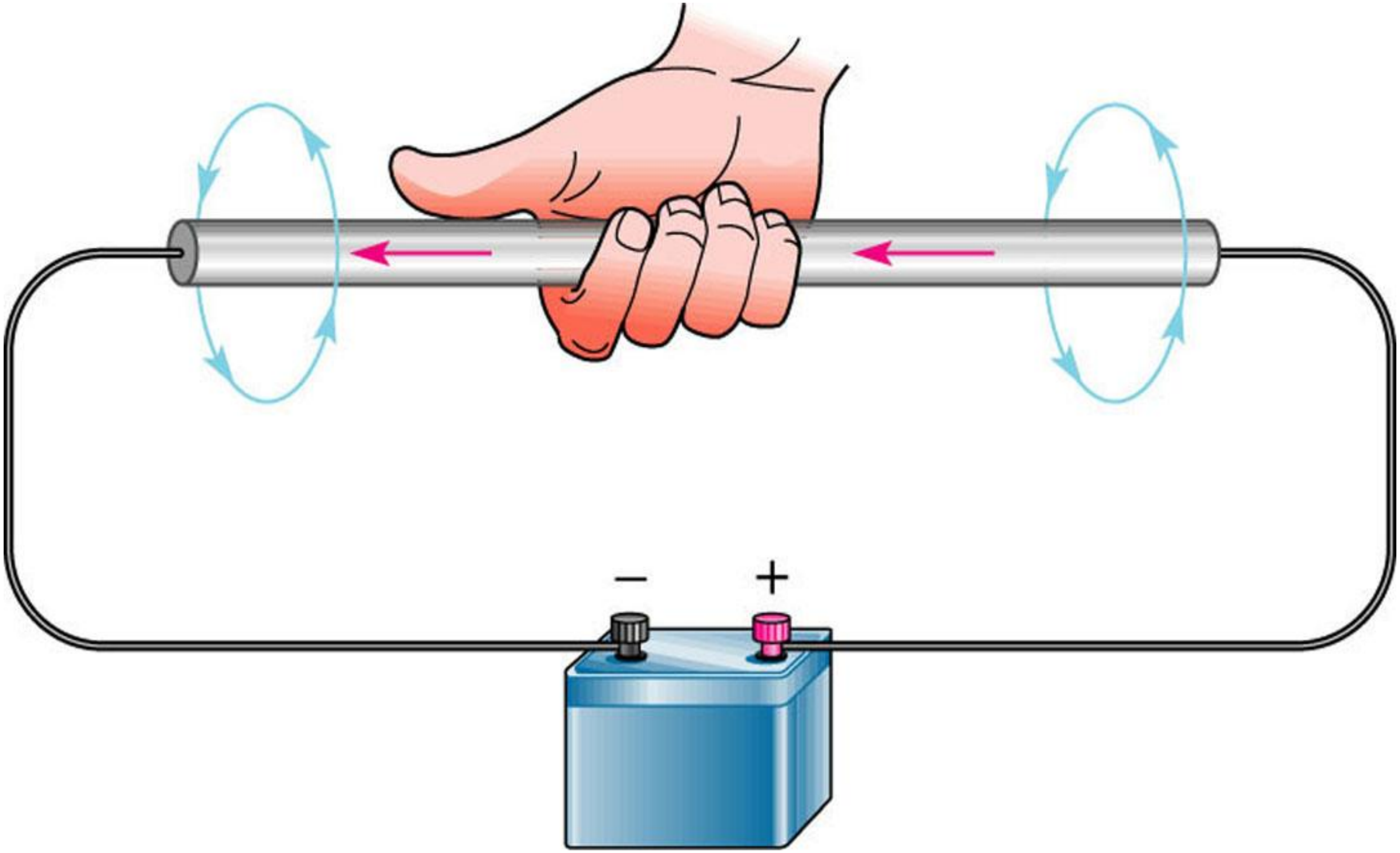
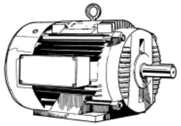


Fig. 9 Illustration of right-hand rule

Electromagnetic Properties

- Permeability (μ)
- The Relative Permeability (μ_r)
- Reluctance, S (R_M)



Permeability (μ)

- Ease with which a magnetic field can be established in a given material is measured by the **permeability** of that material.

- Higher the permeability, a magnetic field can be established easier

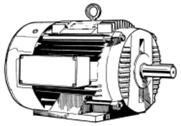
- Symbol μ; its value varies depending on material.

- **μ_o**, permeability of a vacuum is **4π X 10^{-7} Wb/At.m** (weber/ampere-turn.meter) and is used as a reference.

- Ferromagnetic materials typically have;

❖ permeabilities hundreds of times larger than that of a vacuum

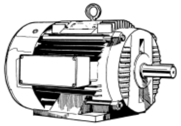
❖ include iron, steel, and their alloys.



The Relative Permeability

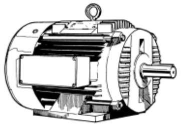
- *(μ_r)* of a material is the ratio of its absolute permeability (μ) to the permeability of a vacuum (μ_o).
- Since μ_r is a ratio, it has no units.

$$\mu_r = \frac{\mu}{\mu_o}$$



Reluctance *(S)*

- Opposition to the establishment of a magnetic field in a material is called reluctance (*S*).
- Value of reluctance is directly proportional to the length ($\mathscr{L}$) of the magnetic path, and inversely proportional to the permeability (μ) and to the c.s.a. (A) of the material;
- $S = \mathscr{L}/\mu A$ (At/Wb)



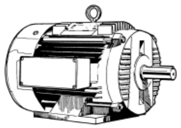
Example 2

- What is the reluctance of a material that has a length of 0.05 m, a cross-sectional area of 0.012 m^2, and a permeability of 3500 μWb/At.m?
- Solution:

$$S = \mathscr{L}/ \mu A$$

$$= 0.05/ (3500 \times 10^{-6}\ \text{Wb/At.m})\ (0.012\text{m}^2)$$

$$= 1190\ \text{At/Wb}$$

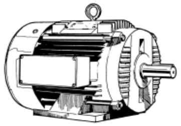


Magnetomotive Force (mmf)

- Current in a conductor produces a magnetic field.
- Force that produces the magnetic field is called the magnetomotive force (mmf).
- Unit of mmf, (At), is established on the basis of the current in a single loop (turn) of wire.
- Formula for mmf is:

$$Fm = NI$$

Fm is the magnetomotive force, N is the no. of turns of wire, I is the current in amperes.



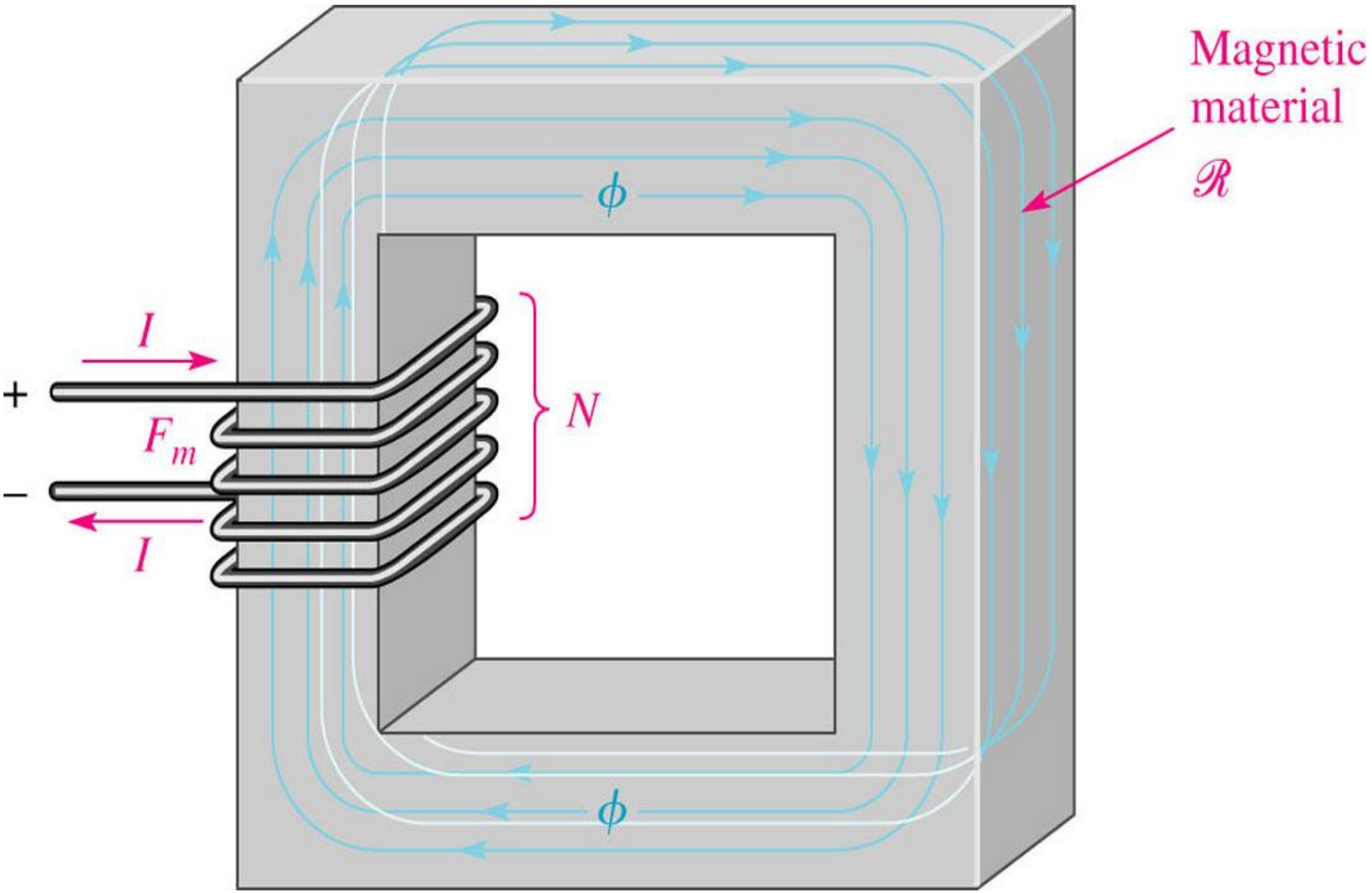
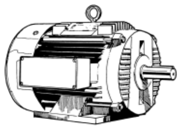


Figure 10 A basic magnetic circuit

Ohm's law for magnetic circuits

- The amount of flux depends on the magnitude of the mmf and on the reluctance of the material, as expressed by:

$$\Phi = \frac{Fm}{R}$$



Example 3

- How much flux is established in the magnetic path of Fig. 12 if the reluctance of the material is 0.28×10^5 At/WB?

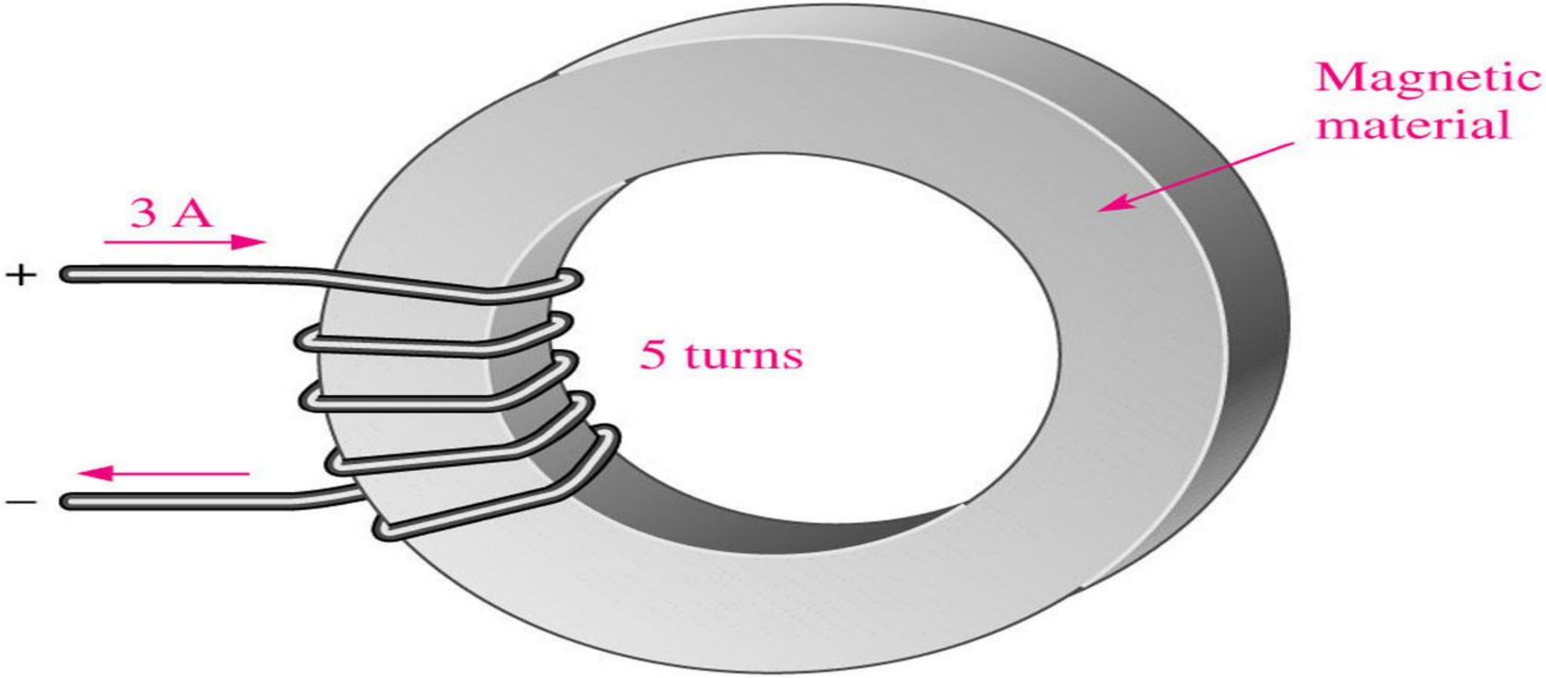
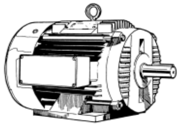


Figure 11

LECTURE TWO

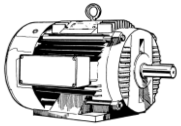


Solution to Example 3

- $$\Phi = Fm/R = NI/R = \frac{(5\text{ t})\,(3\text{ A})}{0.28 \times 10^{5}\text{ At/Wb}}$$

$$= 5.36 \times 10^{-4}\text{ Wb}$$

$$= 536\,\mu\text{Wb}$$



Example 4

- There are two amperes of current through a wire with 5 turns.
- **(a)** What is the mmf?
- **(b)** What is the reluctance of the circuit if the flux is 250 μWb?

Solution

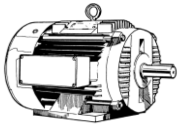
(a) N = 5 and I = 2A

Fm = NI = (5t)(2A) = **10 At**

(b) R = Fm/Φ = 10At/250μWb

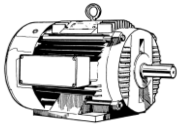
$= 0.04 \times 10^6$ At/Wb

$= 4.0 \times 10^4$ At/Wb



The Electromagnet

- A basic electromagnet is simply a coil of wire wound around a core material that can be easily magnetised.
- The shape of the electromagnet can be designed for various applications.



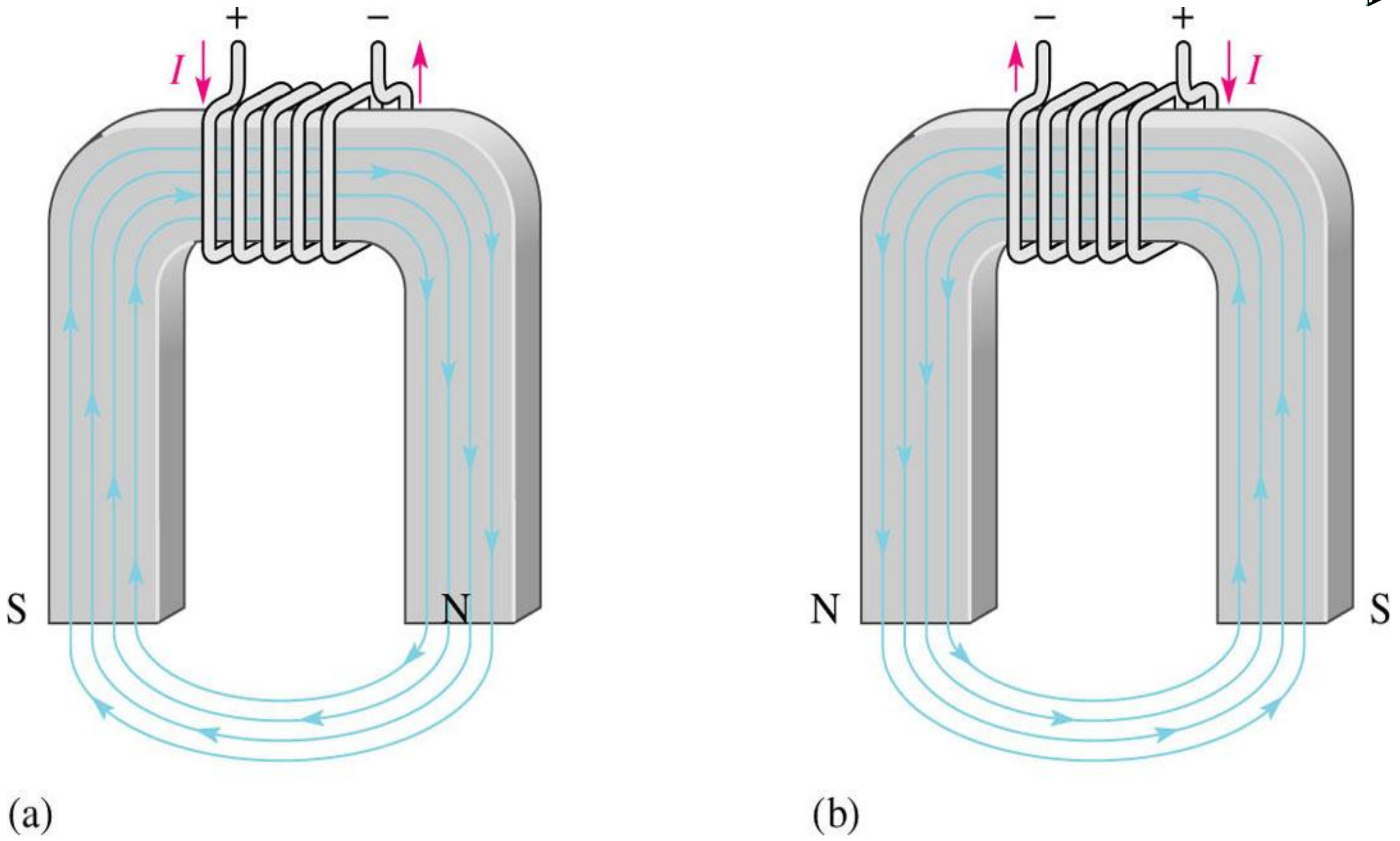
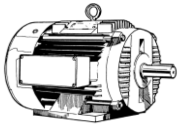


Figure 12 Reversing the current in the coil causes the electromagnetic field to reverse.

Electromagnetic Devices

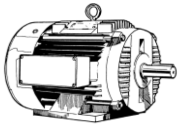
- Magnetic disk/tape read/write head
- Magneto-optical disk
- Transformer
- Solenoid
- Relay
- Speaker

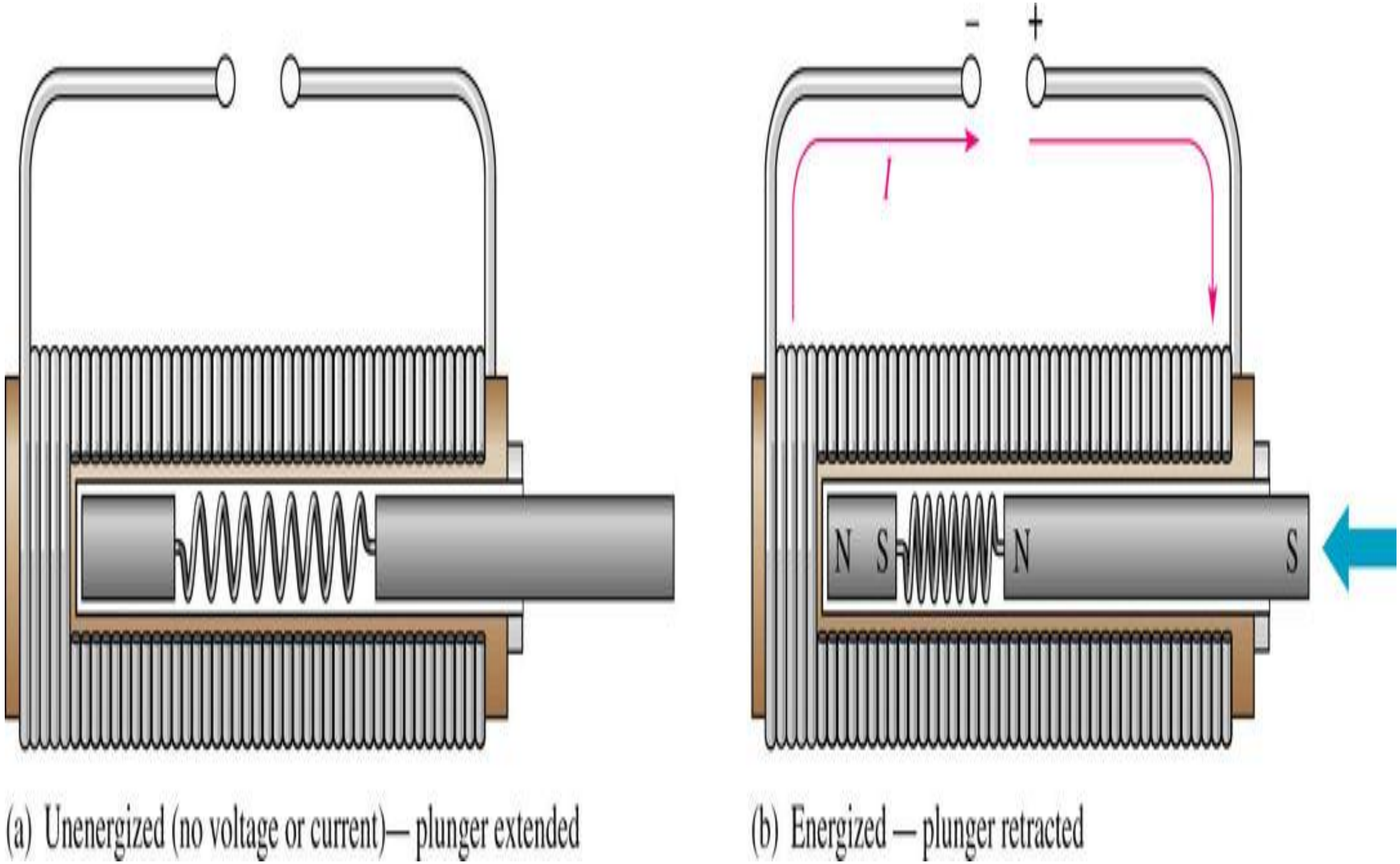


The Solenoid

- Is a type of electromagnetic device that has a movable iron core called *a* ***plunger.***

- Movement of this iron core depends on both an electromagnetic field and a mechanical spring force.





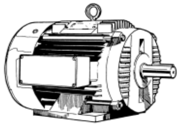
(a) Unenergized (no voltage or current)—plunger extended

(b) Energized—plunger retracted

Figure 13 Basic solenoid operation

The Relay

- Differs from solenoids in that the electromagnetic action is used to open or close electrical contacts rather than to provide mechanical movement.

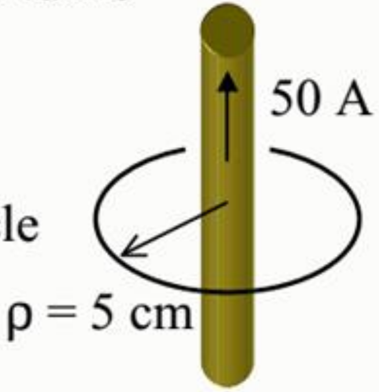


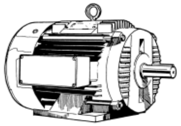
Magnetic Field Intensity

- Whenever a magnetic flux, ϕ, exist in a conductor or component, it is due to the presence of a **magnetic field intensity**, *H*, given by
$$H = \frac{\mathcal{F}}{l}$$
where
  - H = magnetic field intensity [A/m]
  - $\mathcal{F}$ = magnetomotive force [A] or ampere turns
  - l = length of the magnetic circuit [m]
  - example

find the magnetic field intensity at the circle

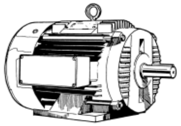
$$H = \frac{U}{2\pi\rho} = \frac{50\,\text{A}}{2\pi(0.05\,\text{m})} = 159\,\text{A/m}$$

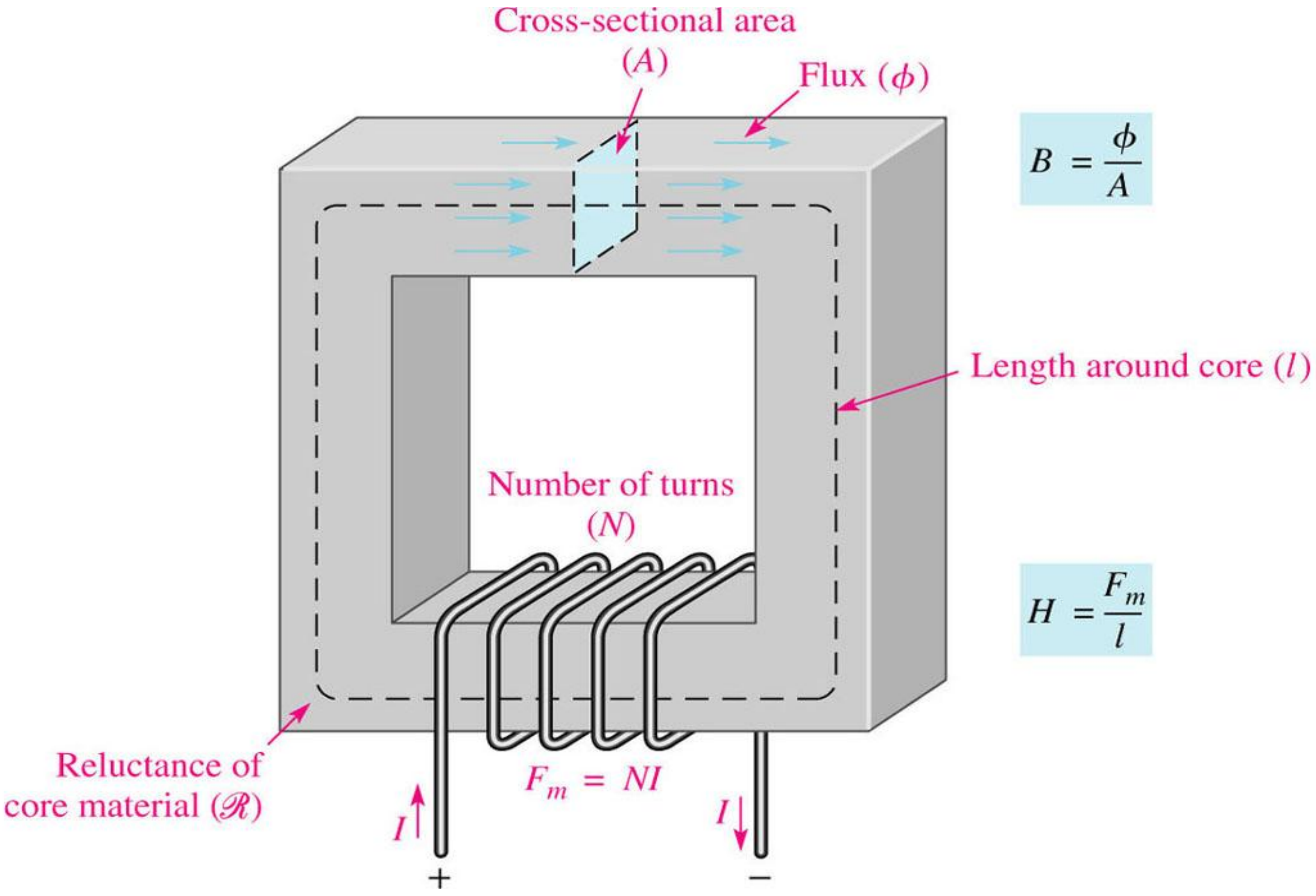




Magnetising Force (H)

- Since $\Phi = Fm/R$, as Fm increases, the flux increases.
- Also, magnetising force (H) increases.
- Recall that; flux density (B) is the flux per unit area ($B = \Phi/A$), so B is also proportional to H.
- Curve showing how these two quantities (B & H) are related is called the B-H curve (hysteresis curve).





$F_m = NI$

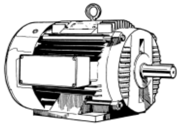
$$B = \frac{\phi}{A}$$

$$H = \frac{F_m}{l}$$

Fig. 14 Parameters that determine the magnetising force (*H*) and the flux density (*B*).

The Hysteresis Curve

- ***Hysteresis*** is a characteristic of a magnetic material whereby a change in magnetisation lags the application of a magnetising force.
- Magnetising force (H) can be increased or decreased by varying the current through the coil of wire (reversed by reversing the voltage polarity across the coil).



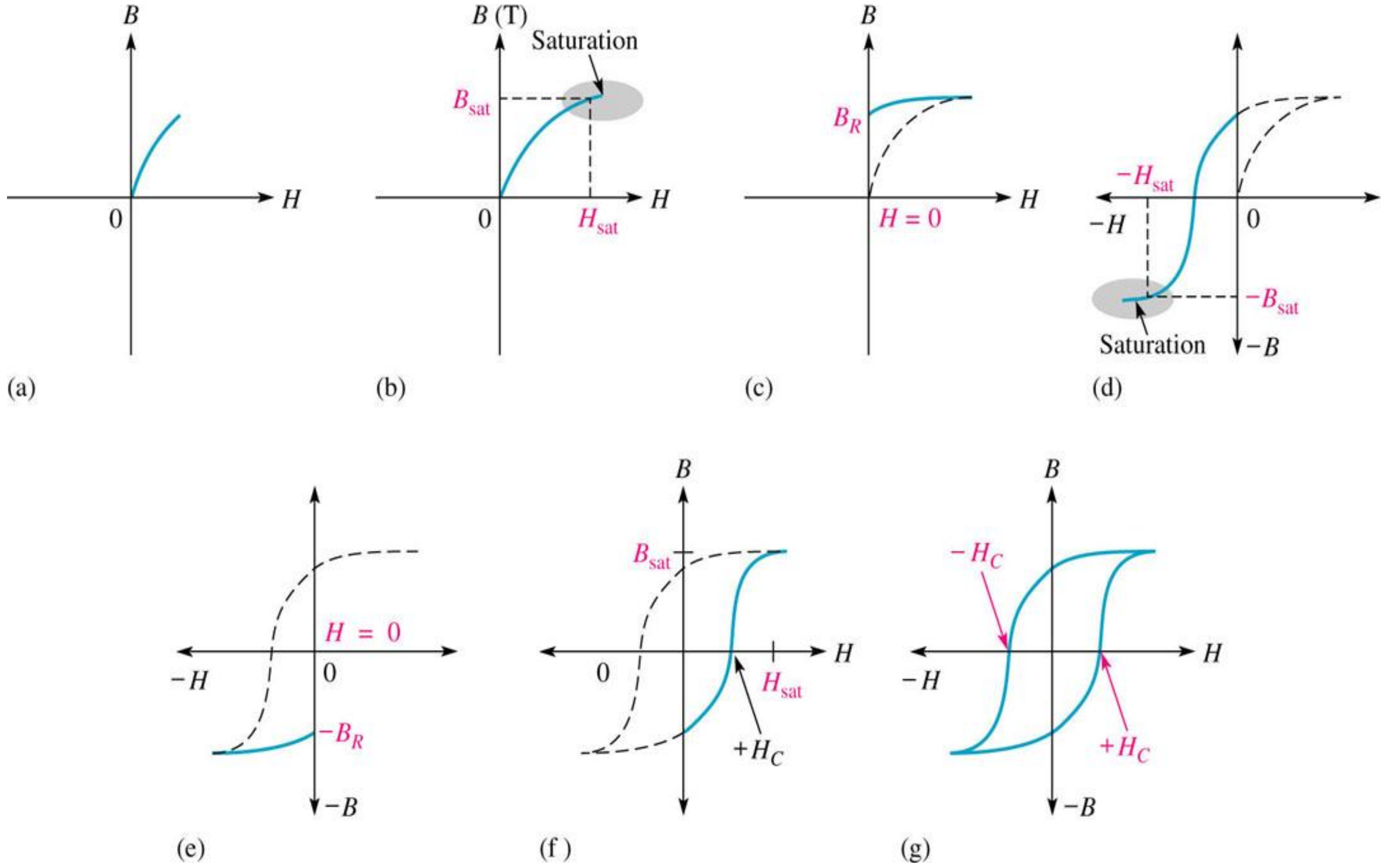
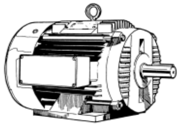


Fig 15 Development of a magnetic hysteresis curve



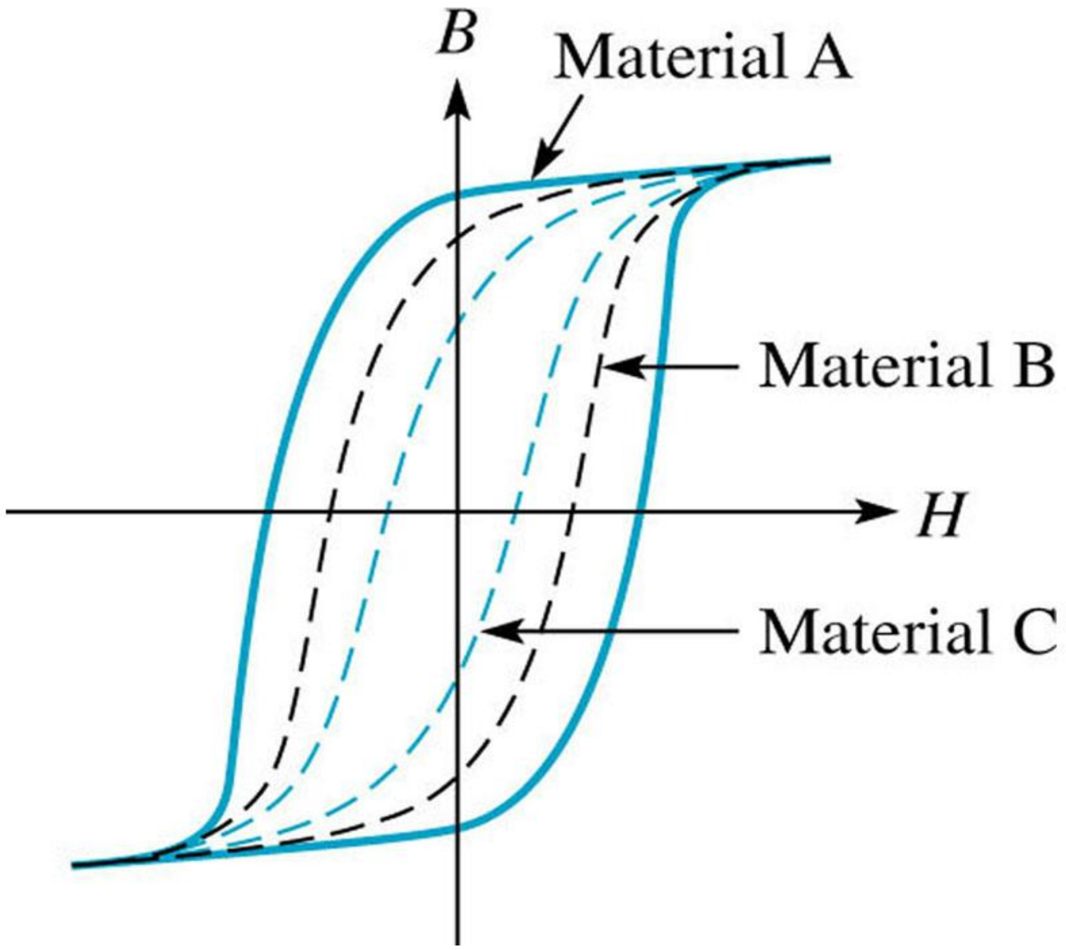
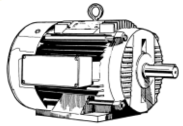


Fig: 16(g) Complete B-H Curve ~ The Hysteresis Curve

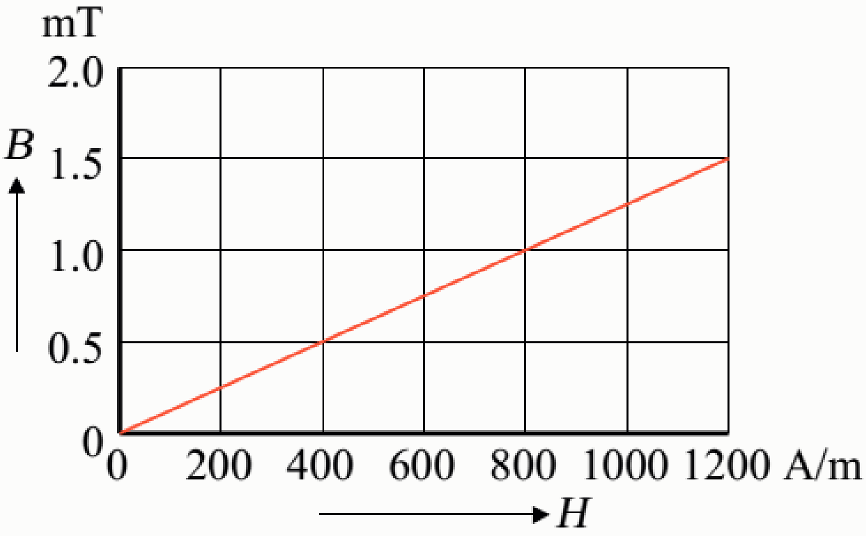


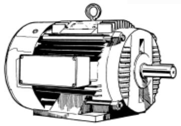
Relationship between B-H in Free Space

- In free space, the magnetic flux density B is directly proportional to the magnetic field intensity H
- The constant of proportionality for free space is called the permeability constant, μ_0

$$B = \mu_0 H$$

  - in the SI system, $\mu_0 = 4\pi\times10\text{-}7$ H/m [henry/meter]



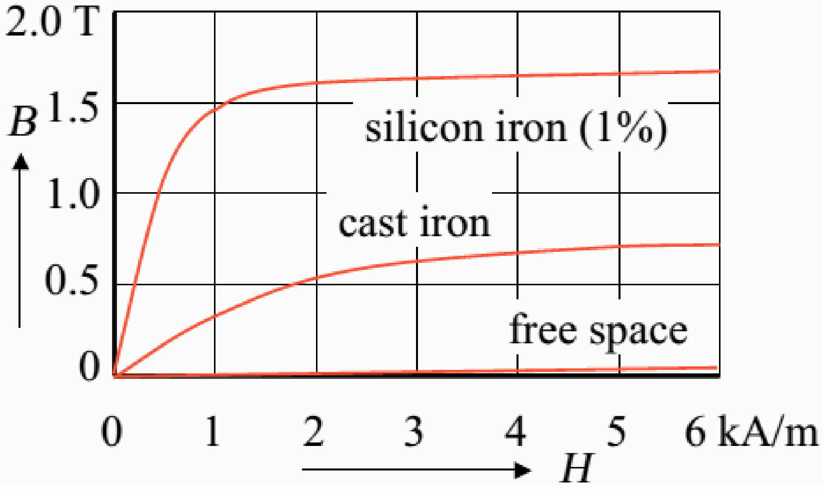


B-H Characteristic of Magnetic Material

- The flux density is influenced by the magnetic property of the material in which the flux passes
  - instead of specifying a permeability for every material, a relative permeability is defined, $\mu_r = \mu / \mu_0$
  - relative permeability is unitless

$$B = \mu_0 \, \mu_r \, H$$

- for many materials, the relative permeability is not constant but varies nonlinear w.r.t. B

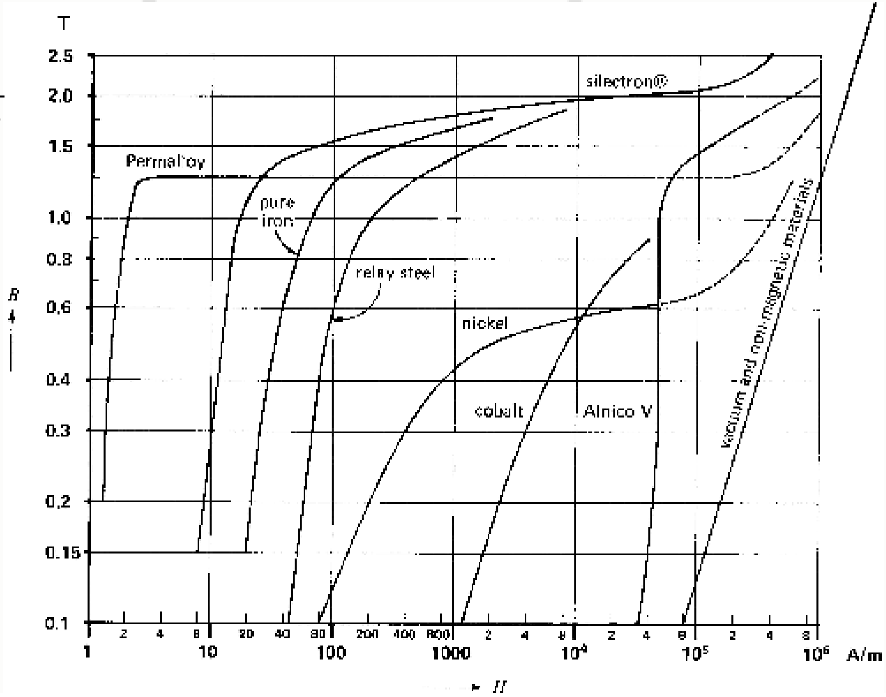


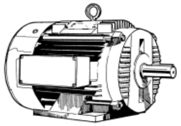
Determining Relative Permeability

- One can find the relative permeability in a material by taking the ratio of the flux density in the material to the flux density that would have been produced in free-space

$$\mu_r = \frac{B}{\mu_0 H} \approx 800{,}000 \frac{B}{H}$$

  - Example
    Determine the relative permeability of relay steel (1% silicon) at a flux density of 0.6 T and 1.4 T



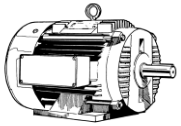


Example 6

- A mild steel ring of c.s.a. 4 cm has a radial air-gap of 3 mm cut into it. If the mean length of the mild steel path is 300 mm.

Calculate the magnetomotive force to produce a flux of 0.48 mWb.

(Use B-H curve on page 78)



Solution to Example 6

- Two parts to the circuit - mild steel and the air-gap
- **For the mild steel:**

$$B = \Phi/A$$

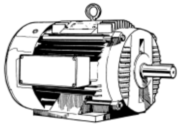
$$\mathbf{= 0.48 \times 10^{-3} / 4 \times 10^{-4} = 1.2\ T}$$

(From B-H curve for mild steel on p78)

when B = 1.2 T, H = 1800 A/m (or close)

- Hence, m.m.f. for the mild steel

$$Hl = (1800)(300 \times 10^{-3}) = \mathbf{540\ A}$$



Solution to Example 6 (cont.d)

For the air-gap:

- The flux density will be the same in the air-gap as in the mild steel, i.e. 1.2 T
- For air, $B = \mu_0 H$ from which,

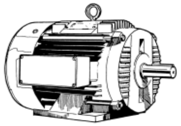
$$H = B/\mu_0$$
$$= 1.2T/4\pi \times 10^{-7}$$
$$\mathbf{= 954930\ A/m}$$

Hence the m.m.f. for the air-gap = Hl

$$= (954930)(3 \times 10^{-3})$$
$$\mathbf{= 2865\ A}$$

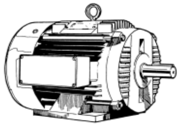
Total m.m.f. to produce a flux of 0.48 mWb

$$= 540 + 2865 = \mathbf{3405\ A}$$



Materials with a low Retentivity

- Do not retain a magnetic field very well while those with high retentivities exhibit values of B_R very close to the saturation value of B.
- Retentivity in a magnetic material can be an advantage or a disadvantage.
- In permanent magnets and memory cores ~ high retentivity is required.
- In ac motors ~ retentivity is undesirable

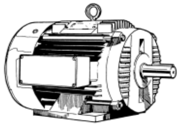


Electromagnetic Induction

- Relative motion between a conductor and a magnetic field, a voltage is produced across the conductor.

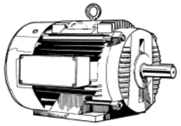
- Resulting voltage is an induced voltage.

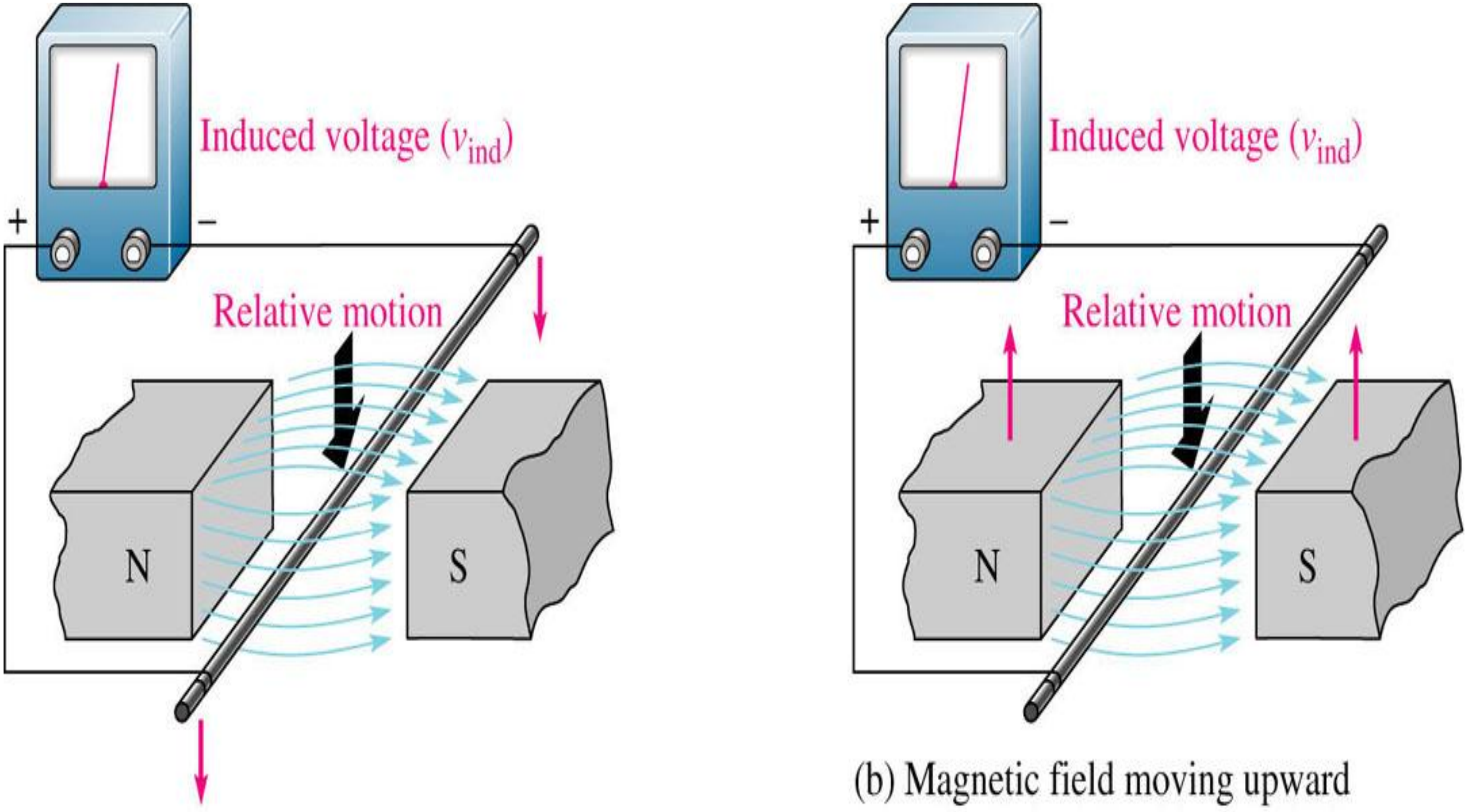
- Transformers, electrical generators, electrical motors, and many other devices possible.



Relative Motion

- When a wire is moved across a magnetic field, there is a relative motion between the wire and the magnetic field.
- Likewise, when a magnetic field is moved past a stationary wire, there is also relative motion.
- In either case, this relative motion results in an ***induced voltage*** (v_{ind}) .





(a) Wire moving downward

(b) Magnetic field moving upward

Fig. 17 Relative motion between a wire and a magnetic field

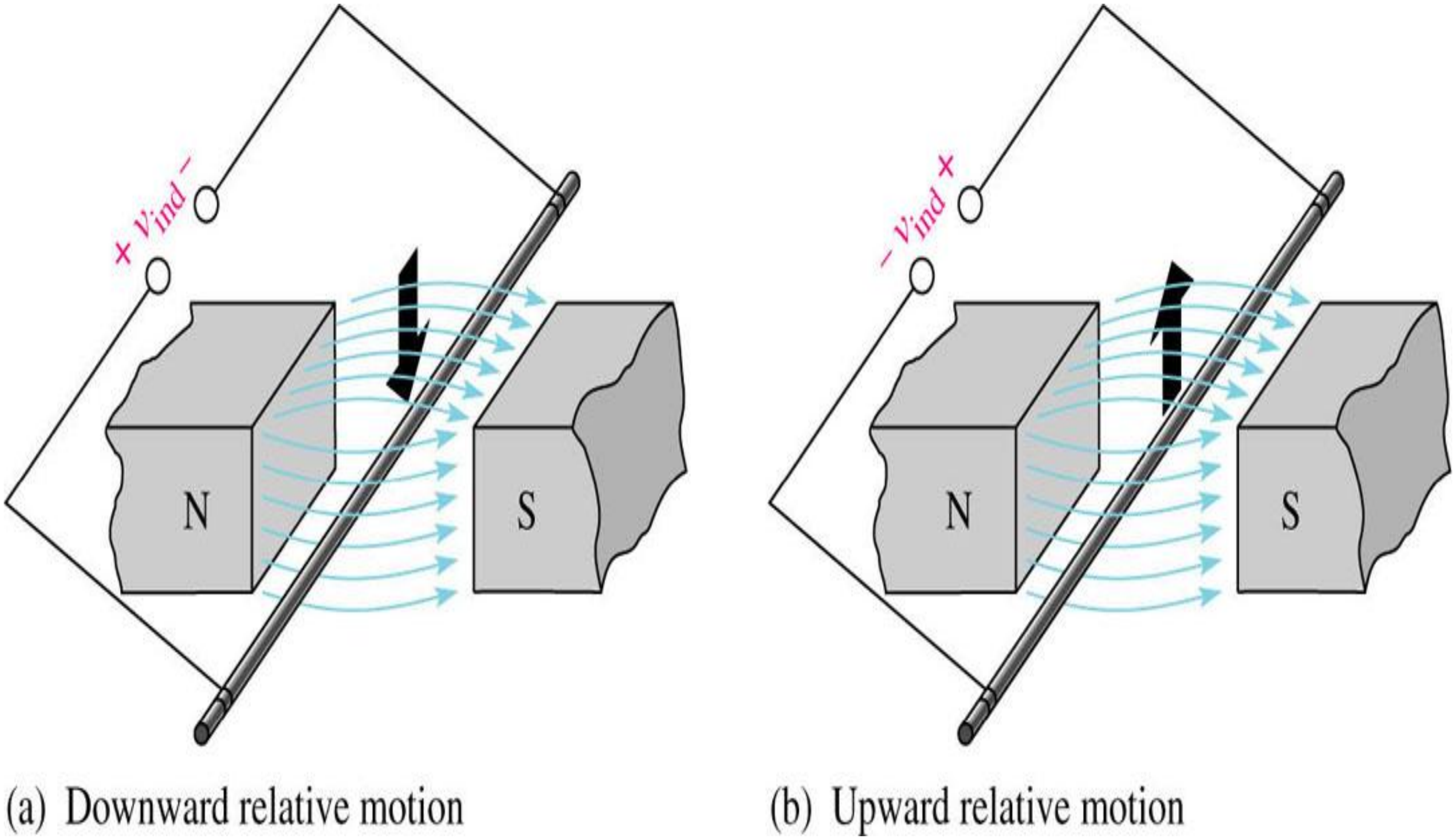


Fig. 18 Polarity of induced voltage depends on direction of motion.

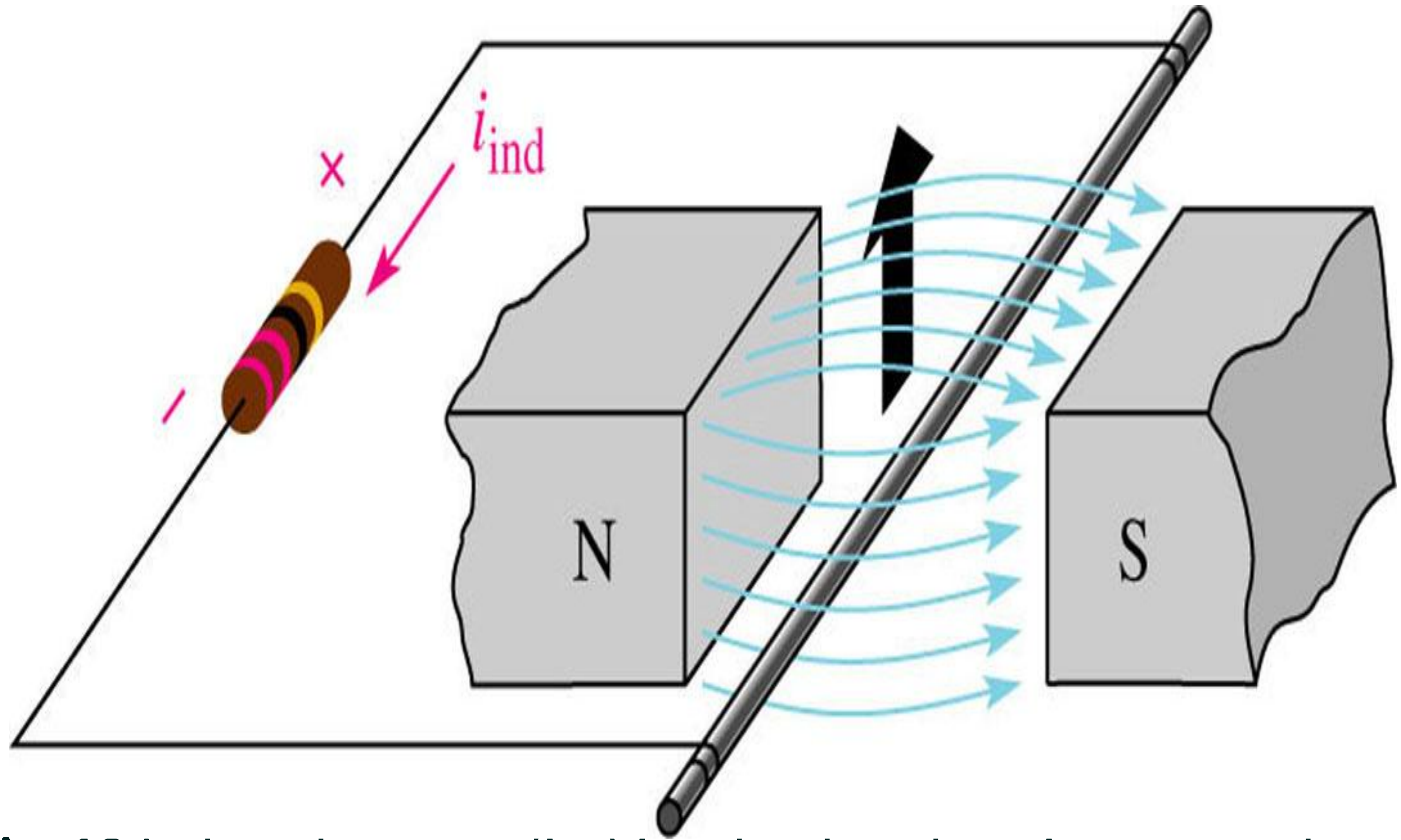
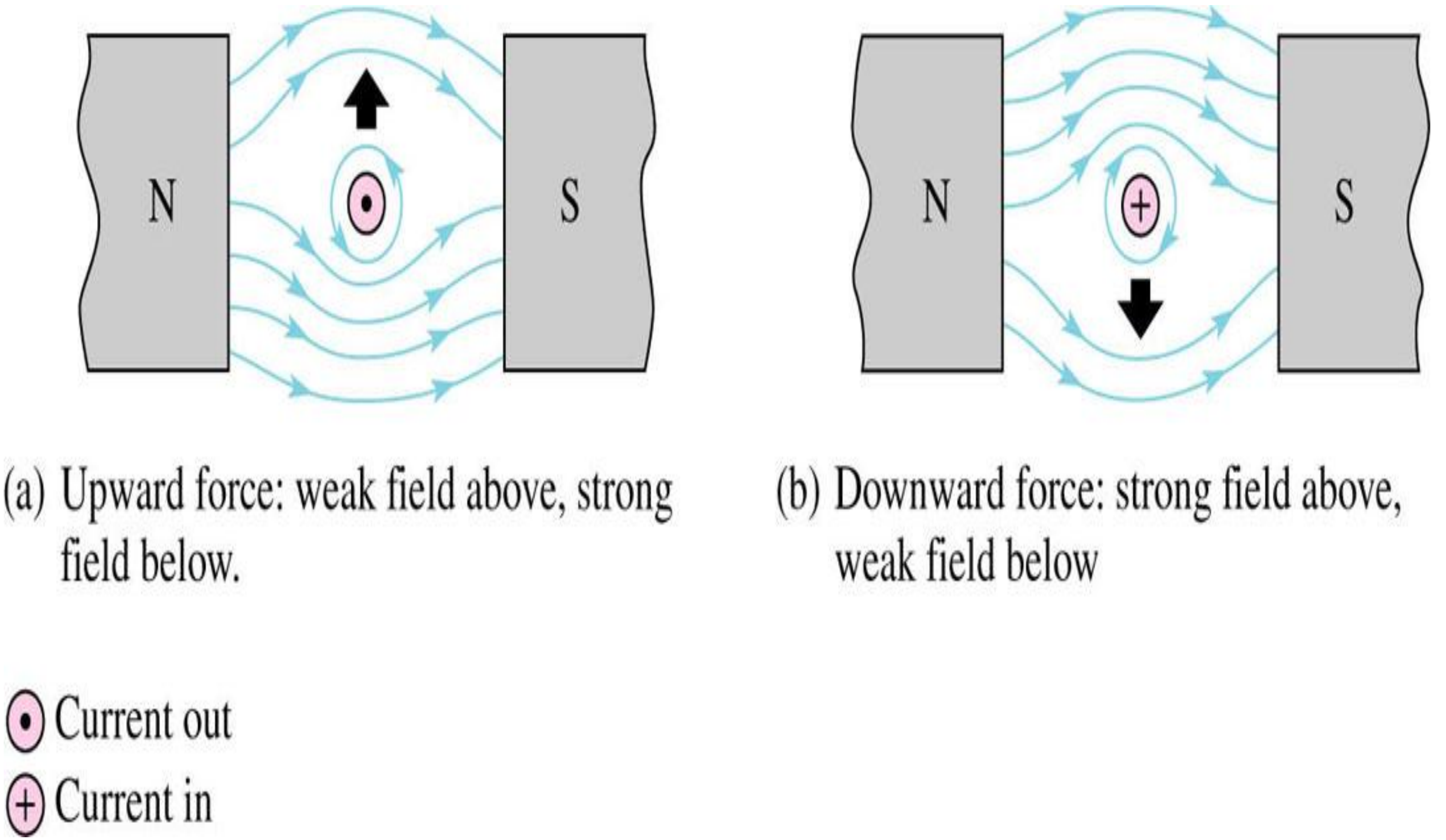


Fig. 19 Induced current (i_{ind}) in a load as the wire moves through the magnetic field.



(a) Upward force: weak field above, strong field below.

(b) Downward force: strong field above, weak field below

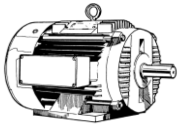
Fig. 20 Forces on a current-carrying conductor in a magnetic field (motor action).

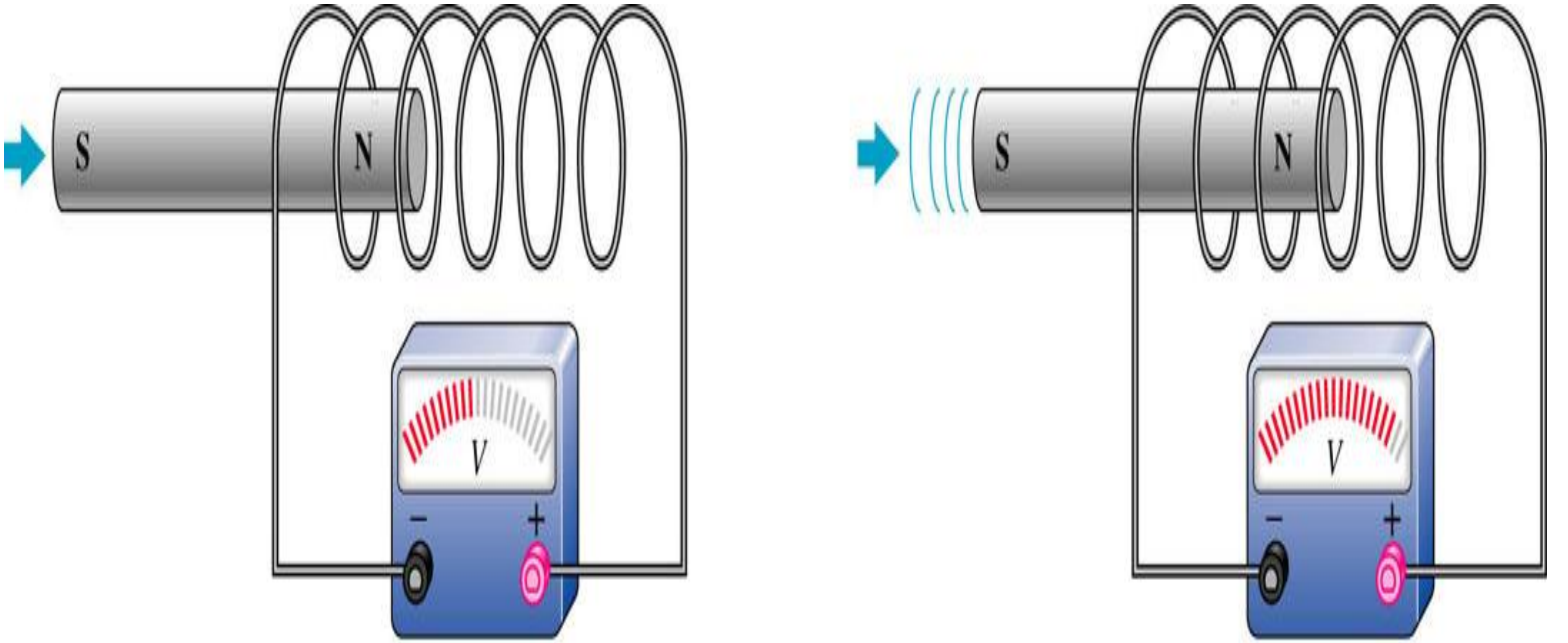
Faraday’s Law

- Michael Faraday discovered the principle of **electromagnetic induction** in 1831.
- **Faraday's two observations:**

(1) The amount of voltage induced in a coil is directly proportional to the rate of change of the magnetic field w.r.t. the coil.

(2) The amount of voltage induced in a coil is directly proportional to the no. of turns of wire in the coil.

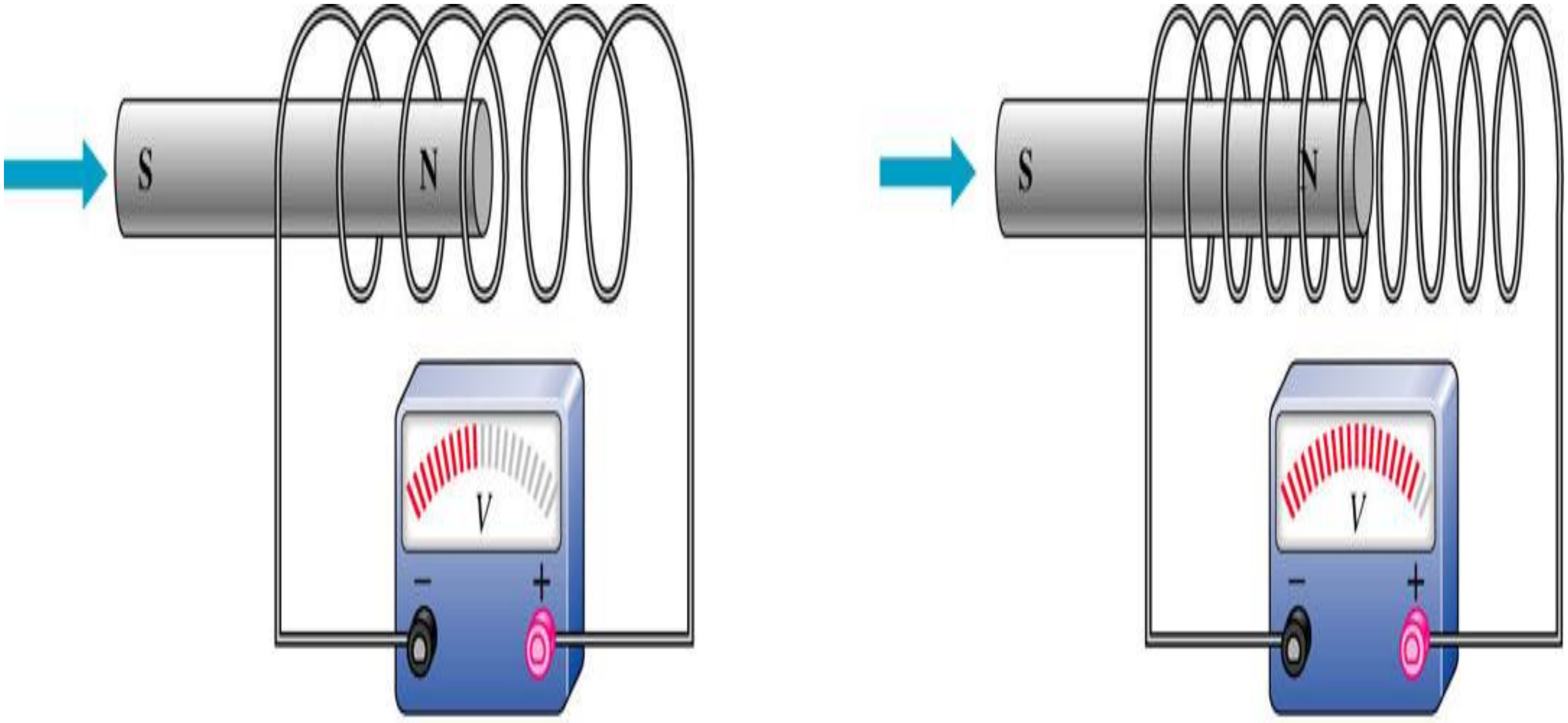




(a) As the magnet moves slowly to the right, its magnetic field is changing with respect to coil, and a voltage is induced.

(b) As the magnet moves more rapidly to the right, its magnetic field is changing more rapidly with respect to coil, and a greater voltage is induced.

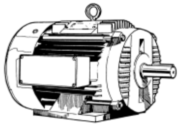
Fig. 21 A demonstration of Faraday's first observation: The amount of induced voltage is directly proportional to the rate of change of the magnetic field w.r.t. the coil.



(a) Magnet moves through a coil and induces a voltage.

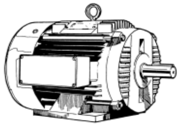
(b) Magnet moves at same rate through a coil with more turns (loops) and induces a greater voltage.

Fig. 22 A demonstration of Faraday’s second observation: The amount of induced voltage is directly proportional to the no. of turns in the coil



Faraday’s Law

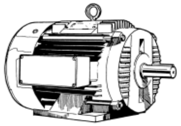
- **The voltage induced across a coil of wire equals the number of turns in the coil times the rate of change of the magnetic flux.**



Lenz’s Law

- Defines the polarity or direction of the induced voltage.

- **When the current through a coil changes, the polarity of the induced voltage created by the changing magnetic field is such that it always opposes the change in current that caused it.**

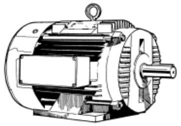


Faraday’s Law

- Electromagnetic induction
  - If the flux linking a loop (or turn) varies as a function of time, a voltage is induced between its terminals
  - The value of the induced voltage is proportional to the rate of change of flux
- In SI units,

$$E = N\frac{\Delta\Phi}{\Delta t}$$

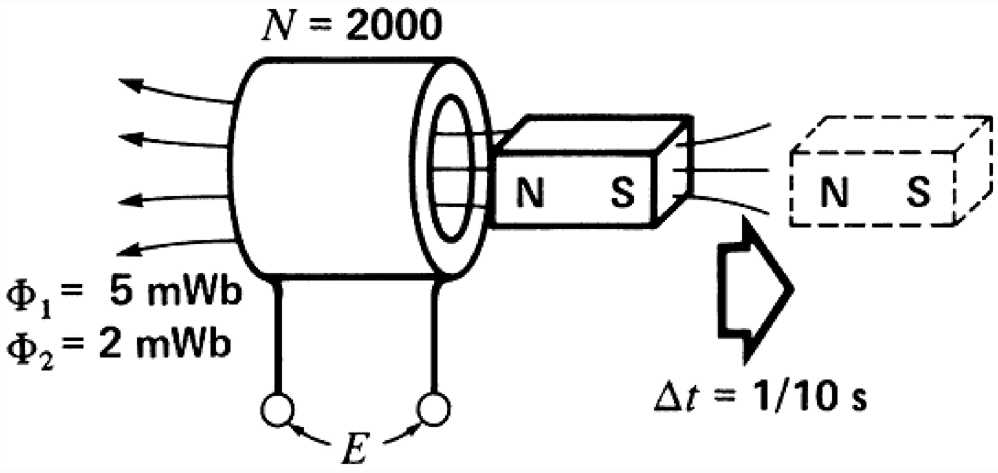
where
  - E = induced voltage [V]
  - N = number of turns in the coil
  - $\Delta\Phi$ = change of flux inside the coil [Wb]
  - Δt = time interval of the flux changes [s]

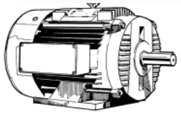


Faraday’s Law

– Example

- a coil of 2000 turns surrounds a flux of 5 mWb produced by a permanent magnet
- the magnet is suddenly withdrawn causing the flux inside the coil to drop uniformly to 2 mWb in 0.100 s
- what is the induced voltage?





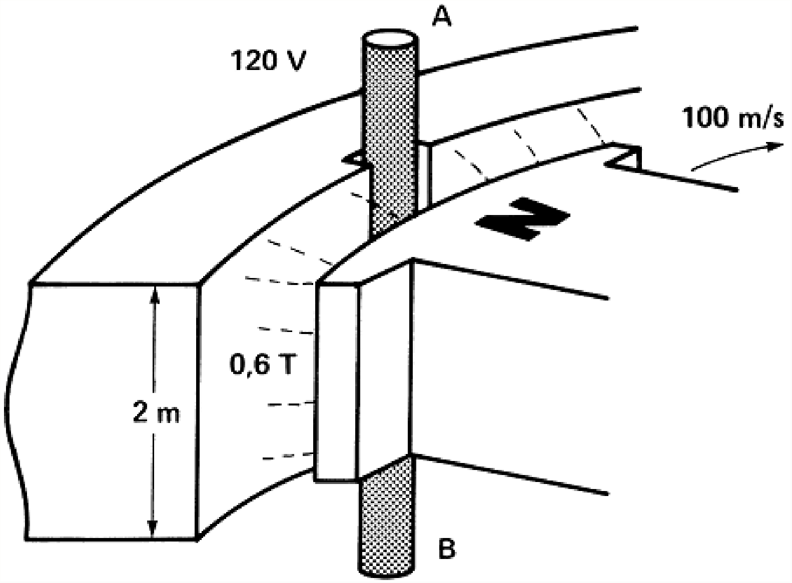
Voltage Induced in a Conductor

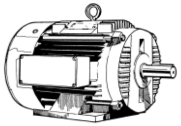
- It is often easier to calculate the induced voltage on a segment of conductor instead of the voltage on a coil

$$E = B\,l\,v$$

where

- E = induced voltage [V]
- B = flux density [T]
- l = active length of conductor in the magnetic field [m]
- v = relative speed of the conductor [m/s]

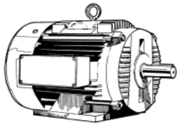




Lorentz Force

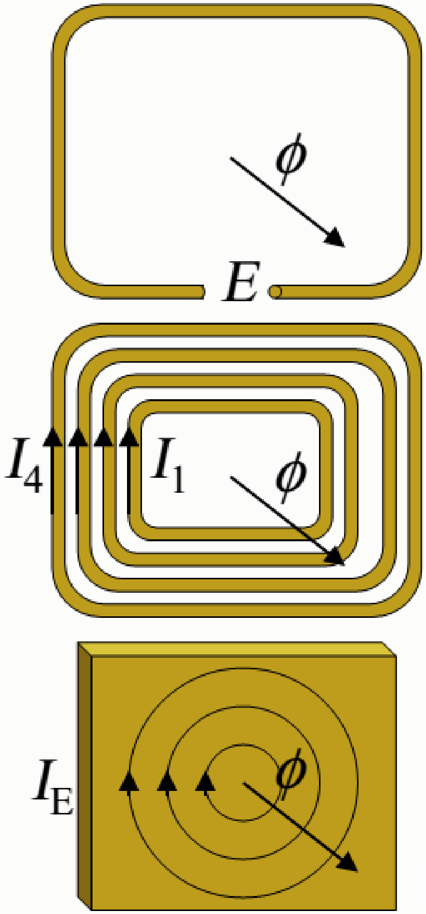
- A current-carrying conductor sees a force when placed in a magnetic field
  - fundamental principle for the operation of motors
  - the magnitude of the force depends upon orientation of the conductor with respect to the direction of the field
  - force is greatest when the conductor is perpendicular to the field
- The Lorentz or electromagnetic force: $F = B\,l\,I \sin\theta$

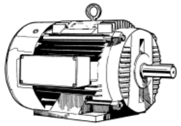
  where
  - F = force acting on the conductor [N]
  - θ = angle between the flow directions of current and flux



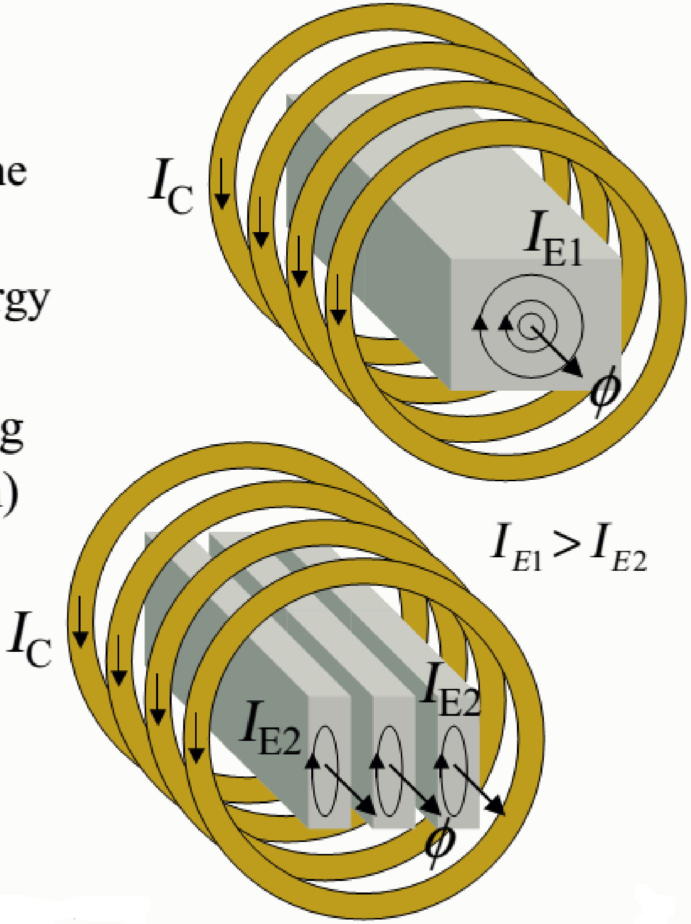
Eddy Currents

- An ac flux ϕ linking a rectangular-shaped conductor induces an ac voltage E across its terminals
- If the conductor terminals are shorted, a substantial current flows
- The same flux linking smaller coils induce lesser voltages and lower currents
- A solid metal plate is basically equivalent to a densely packed set of rectangular-shaped coils
- The induced currents flowing inside the plate are eddy currents, and flow to oppose the change in flux

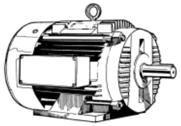




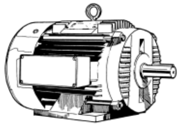
Eddy Current Losses



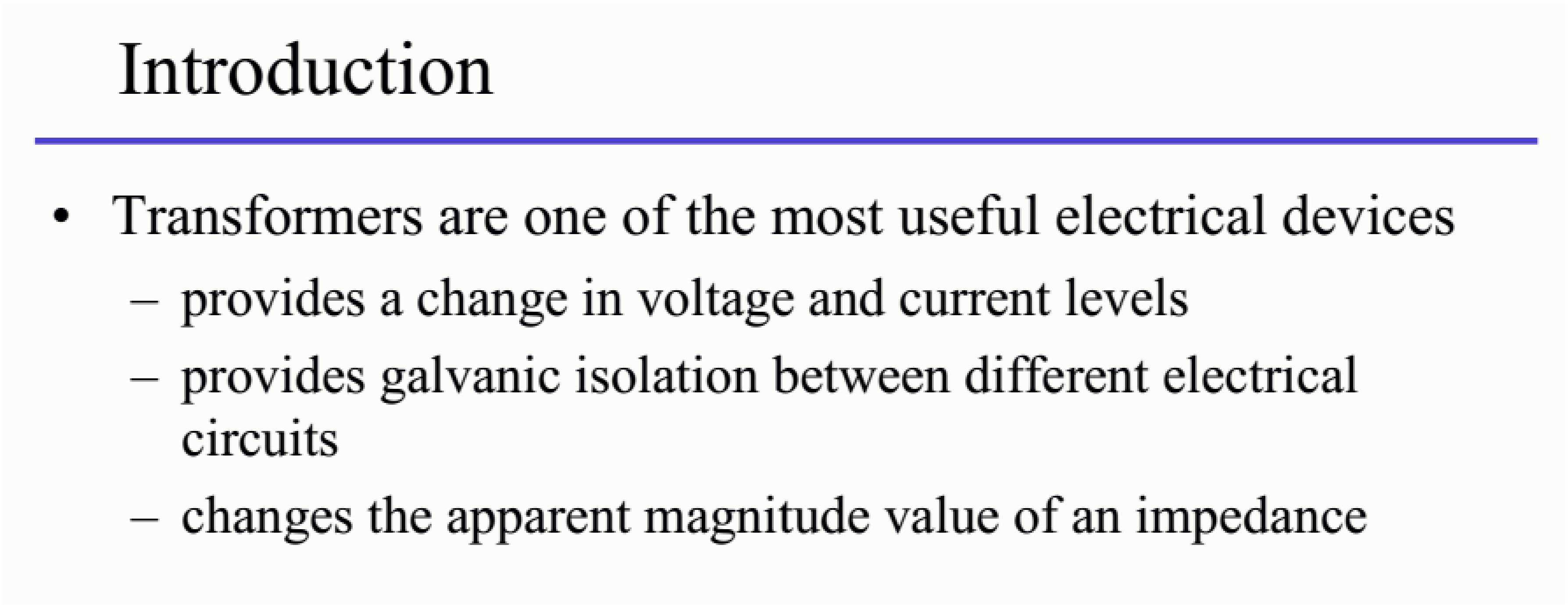
- Eddy currents become a problem when iron must carry an ac flux
  - eddy currents flow throughout the entire length of the iron core
  - resistance in the iron causes energy to be dissipated as heat
- Losses can be reduced by splitting the core into sections (lamination)
  - subdividing causes the losses to decrease progressively
  - varnish coatings insulate the laminates from current flows
  - silicon in the iron increases the resistance



LECTURE THREE

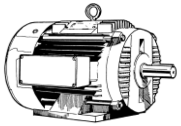


Transformer



Introduction

- Transformers are one of the most useful electrical devices
  - provides a change in voltage and current levels
  - provides galvanic isolation between different electrical circuits
  - changes the apparent magnitude value of an impedance

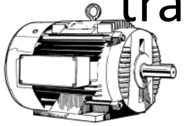


Some history

Historically, the first electrical power distribution system developed by Edison in 1880s was transmitting DC. It was designed for low voltages (safety and difficulties in voltage conversion); therefore, high currents were needed to be generated and transmitted to deliver necessary power. This system suffered significant energy losses!

The second generation of power distribution systems (what we are still using) was proposed by Tesla few years later. His idea was to generate AC power of any convenient voltage, step up the voltage for transmission (higher voltage implies lower current and, thus, lower losses), transmit AC power with small losses, and finally step down its voltage for consumption. Since power loss is proportional to the square of the current transmitted, raising the voltage, say, by the factor of 10 would decrease the current by the same factor (to deliver the same amount of energy) and, therefore, reduce losses by factor of 100.

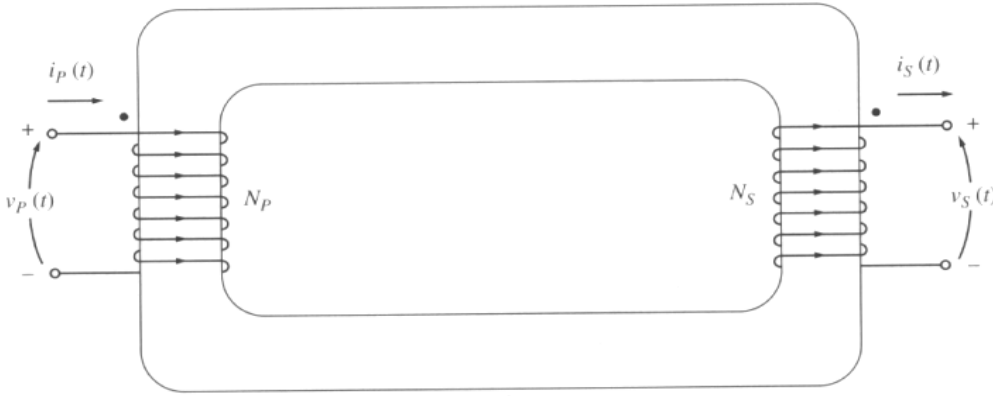
The step up and step down voltage conversion was based on the use of transformers.



Preliminary considerations

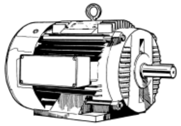
A transformer is a device that converts one AC voltage to another AC voltage at the same frequency. It consists of one or more coil(s) of wire wrapped around a common ferromagnetic core. These coils are usually not connected electrically together. However, they are connected through the common magnetic flux confined to the core.

Assuming that the transformer has at least two windings, one of them (primary) is connected to a source of AC power; the other (secondary) is connected to the loads.



The invention of a transformer can be attributed to Faraday, who in 1831 used its principle to demonstrate electromagnetic induction foreseen no practical applications of his demonstration. ☹

Russian engineer Yablochkov in 1876 invented a lighting system based on a set of induction coils, which acted as a transformer.



More history

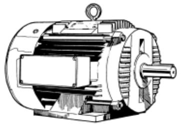
Gaulard and Gibbs first exhibited a device with an open iron core called a 'secondary generator' in London in 1882 and then sold the idea to a company Westinghouse. They also exhibited their invention in Turin in 1884, where it was adopted for an electric lighting system.

In 1885, William Stanley, an engineer for Westinghouse, built the first commercial transformer after George Westinghouse had bought Gaulard and Gibbs' patents. The core was made from interlocking E-shaped iron plates. This design was first used commercially in 1886.

Hungarian engineers Zipernowsky, Bláthy and Déri created the efficient "ZBD" closed-core model in 1885 based on the design by Gaulard and Gibbs. Their patent application made the first use of the word "transformer".

Another Russian engineer Dolivo-Dobrovolsky developed the first three-phase transformer in 1889.

Finally, in 1891 Nikola Tesla invented the Tesla coil, an air-cored, dual-tuned resonant transformer for generating very high voltages at high frequency.



Non Ideal Transformers

- An ideal transformer has no power loss; all power applied to the primary is all delivered to the load. Actual transformers depart from this ideal model. Some loss mechanisms are:

Winding resistance: Causing power to be dissipated in the windings.

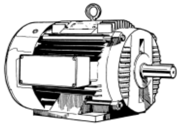
Hysteresis loss: Due to the continuous reversal of the magnetic field.

Core losses: Due to circulating current in the core (eddy currents).

Flux leakage: Flux from the primary that does not link to the secondary.

Winding capacitance: It has a bypassing effect for the windings.

- The ideal transformer does not dissipate power. Power delivered from the source is passed on to the load by the transformer.
- The efficiency of a transformer is the ratio of power delivered to the load (*Pout*) to the power delivered to the primary (*Pin*).



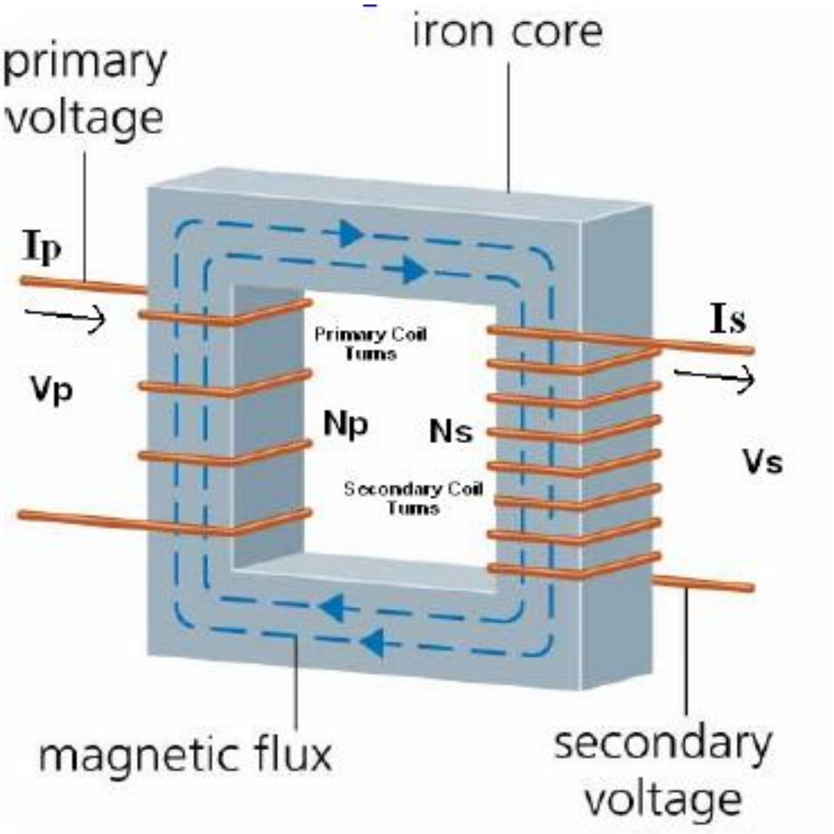
Transformer Principles

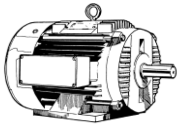
☐ **It has 2 electric circuits called *primary and secondary*.**

☐ **A magnetic circuit provides the link between primary and secondary.**

☐ **When an AC voltage is applied to the primary winding (Vp)of the transformer, an AC current will result (Ip). Ip sets up a time-varying magnetic flux ϕ in the core.**

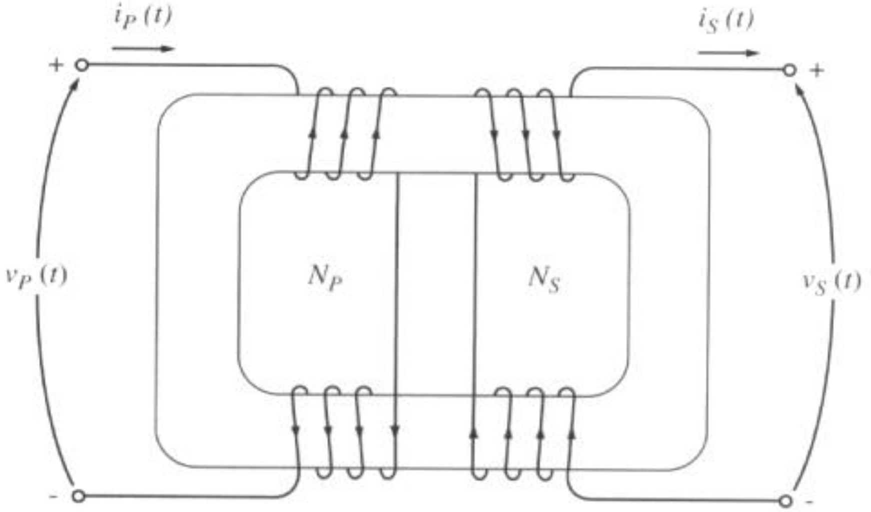
☐ **A voltage is induced to the secondary circuit (Vs) according to the Farday's law.**

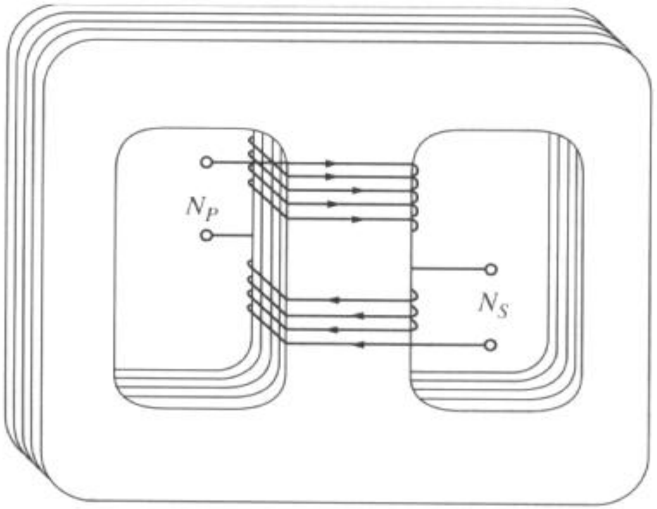




Types and construction

Power transformers

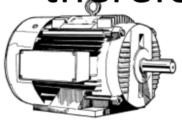




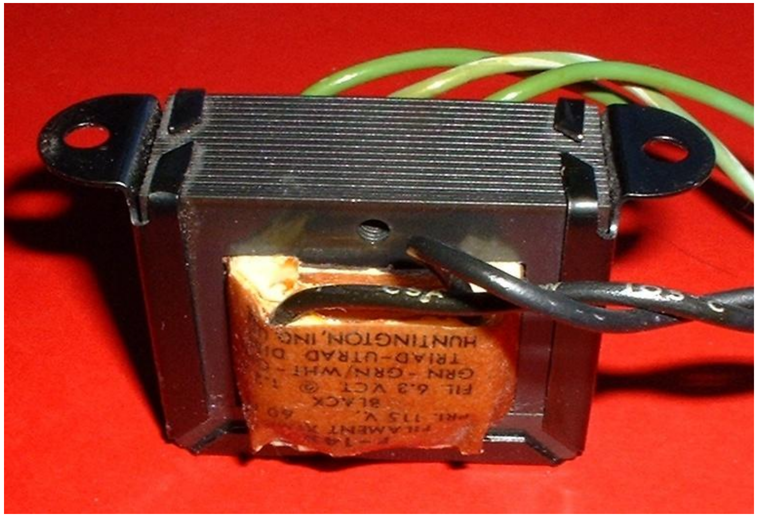
Windings are wrapped around two sides of a laminated square core.

Windings are wrapped around the center leg of a laminated core.

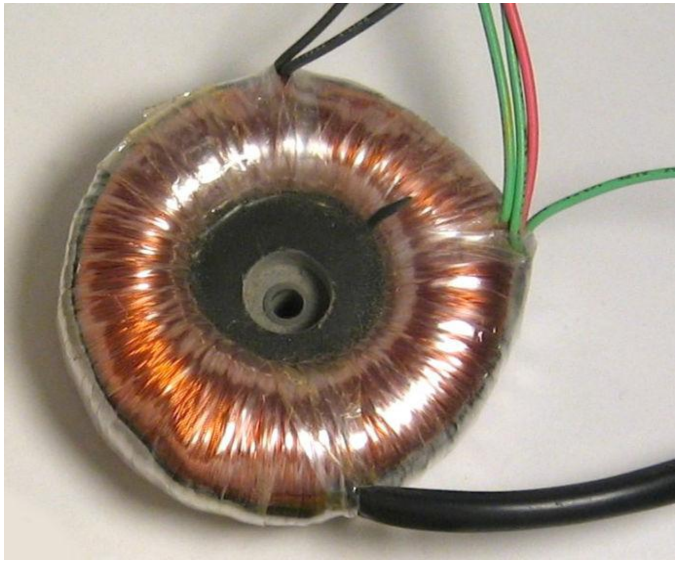
Usually, windings are wrapped on top of each other to decrease flux leakage and, therefore, increase efficiency.



Types and construction



Lamination types



Laminated steel cores

Toroidal steel cores

Efficiency of transformers with toroidal cores is usually higher.

Types and construction

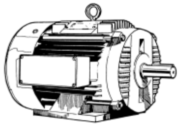
Power transformers used in power distribution systems are sometimes referred as follows:

A power transformer connected to the output of a generator and used to step its voltage up to the transmission level (110 kV and higher) is called a unit transformer.

A transformer used at a substation to step the voltage from the transmission level down to the distribution level (2.3 … 30 kV) is called a substation transformer.

A transformer converting the distribution voltage down to the final level (110 V, 220 V, etc.) is called a distribution transformer.

In addition to power transformers, other types of transformers are used.

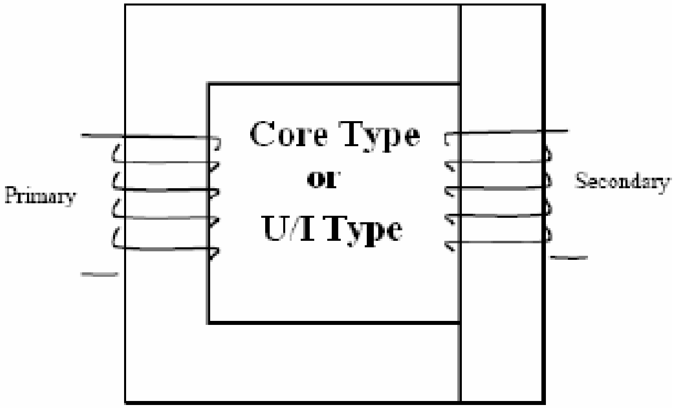


Transformer Construction

2 Type of Transformers:

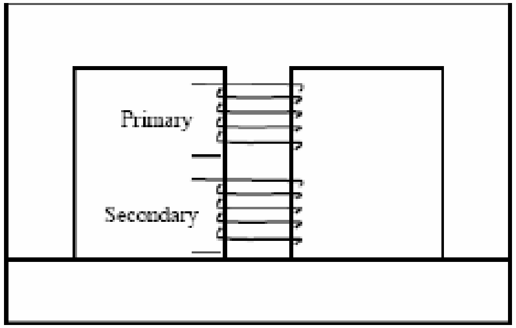
☐ 1- Core (U/I) Type:

is constructed from a stack of U- and I-shaped laminations. In a core-type transformer, the primary and secondary windings are wound on two different legs of the core.

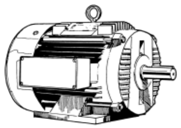


☐ 2- Shell Type: A shell-type

transformer is constructed from a stack of E- and I-shaped laminations. In a shell-type transformer, the primary and secondary windings are wound on the same leg of the core, as concentric windings, one on top of the other one.



Shell Type or E/I type



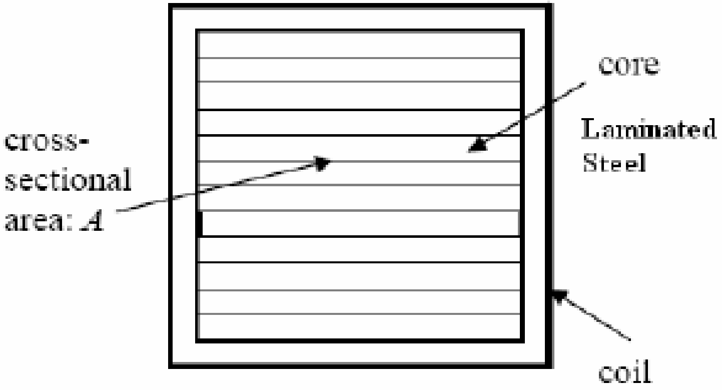
Transformer Core Types

- The magnetic (iron) core is made of thin laminated steel.

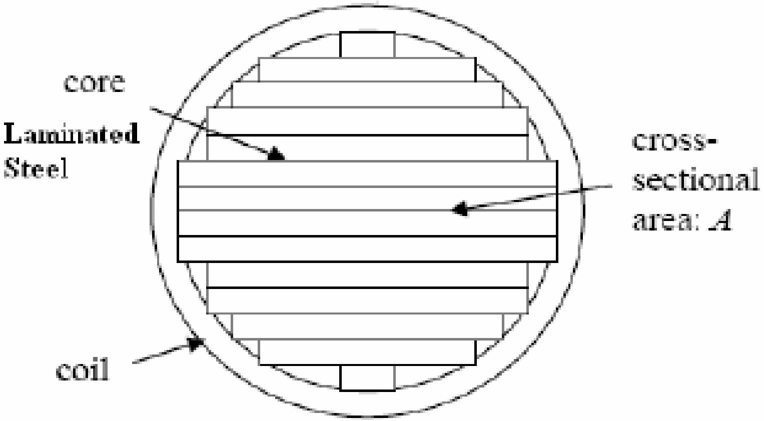
- The reason of using laminated steel is to minimizing the eddy current loss by reducing thickness (t):

$$Pe = kh\,(Bmax\; t\, f)^{2}$$

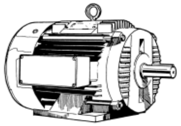
- 2 common cross section of core is square or rectangular) for small transformers and circular (stepped) for the large and 3 phase transformers.



(a) Square core



(b) Stepped core



Ideal Transformers

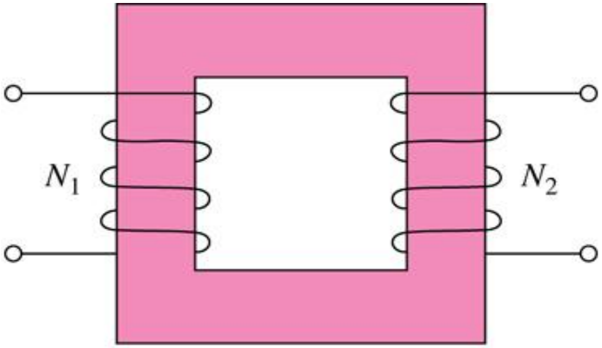
- A n Ideal Transformer is a unity Coupled, lossless transformer in which the primary and secondary coils have infinite self inductances.

A Transformer is ideal if:

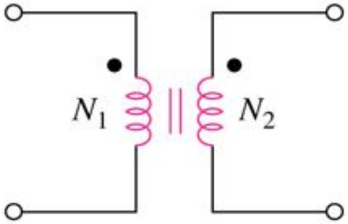
1.) Large reactance coils; $L_1, L_2, M \to \infty$

2.) Unity Coupling k=1.

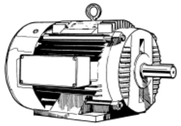
3.) Coils are lossless (R_1=R_2=0)



(a) Ideal transformer

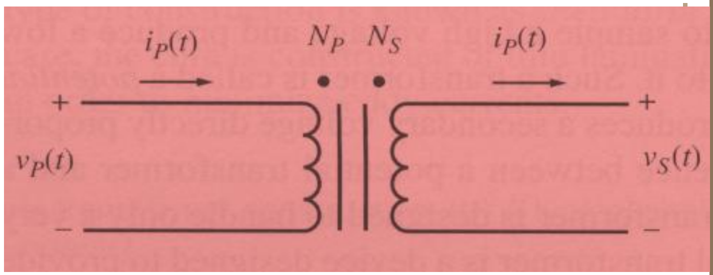


(b) Circuit symbol for the Ideal transformer



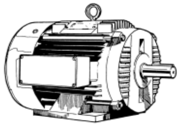
Ideal transformer

One winding's terminal is usually marked by a dot used to determine the polarity of voltages and currents.



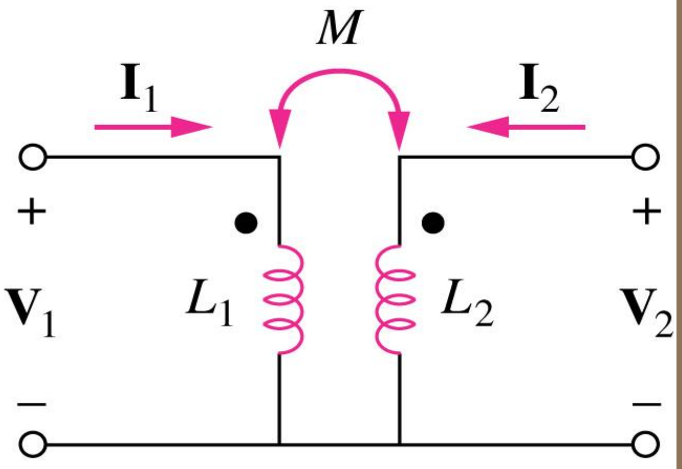
If the voltage is positive at the dotted end of the primary winding at some moment of time, the voltage at the dotted end of the secondary winding will also be positive at the same time instance.

If the primary current flows into the dotted end of the primary winding, the secondary current will flow out of the dotted end of the secondary winding.



Input-Output Variables of an Ideal Transformer

- The input and output voltages and currents of an ideal transformer are related only by the turns ratio.



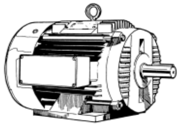
$$V_1 = j\omega L_1 I_1 + j\omega M I_2 \qquad I_1 = \frac{V_1 - j\omega M I_2}{j\omega L_1}$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2 \qquad V_2 = j\omega L_2 I_2 + \frac{M V_1}{L_1} - \frac{j\omega M^2 I_2}{L_1}$$

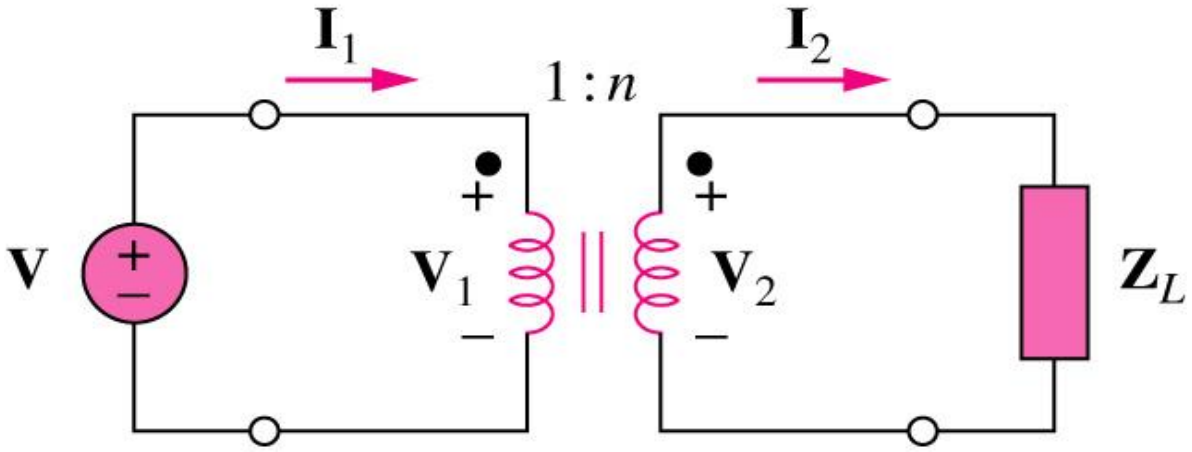
Perfect Coupling $k = 1$, Thus we have $M = \sqrt{L_1 L_2}$ Sunstitute

$$V_2 = j\omega L_2 I_2 + \frac{\sqrt{L_1 L_2} V_1}{L_1} - \frac{j\omega L_1 L_2 I_2}{L_1} = \sqrt{\frac{L_2}{L_1}} V_1 = n V_1 = \frac{N_2}{N_1} V_1$$

$$\boxed{\frac{V_2}{V_1} = \frac{N_2}{N_1} = n = \text{Turns Ratio}}$$

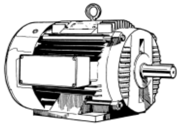


Input-Output Variables of an Ideal Transformer



$$\boxed{\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = n}$$

- A Ideal Transformer is called:
  1.) Step-up transformer if n > 1.
  2.) Step-down transformer if n < 1.
  3.) Isolation transformer if n=1.



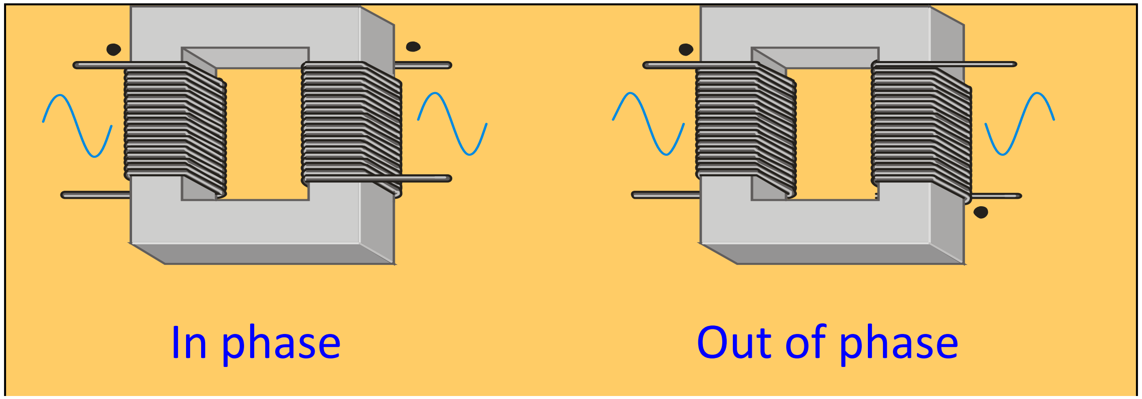
Transformer Dot Convention

- Transformer DOT convention is needed to assign the polarity of the output variables.

  1.) If V_1 and V_2 are BOTH + or BOTH – at the dotted terminals use +n, otherwise –n.

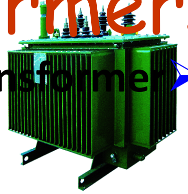
  2.) If I_1 and I_2 BOTH ENTER or BOTH LEAVE the dotted terminals use –n, otherwise +n.

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \pm\frac{N_2}{N_1} = \pm n$$

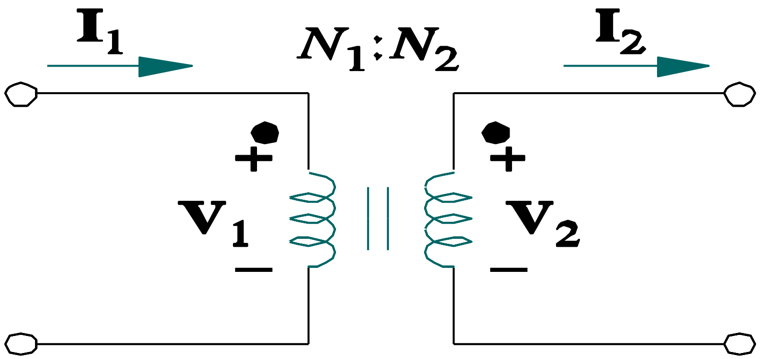


Dot convention indicating the phase relationship between the input and the output.

Dot Convention for Ideal Transformers

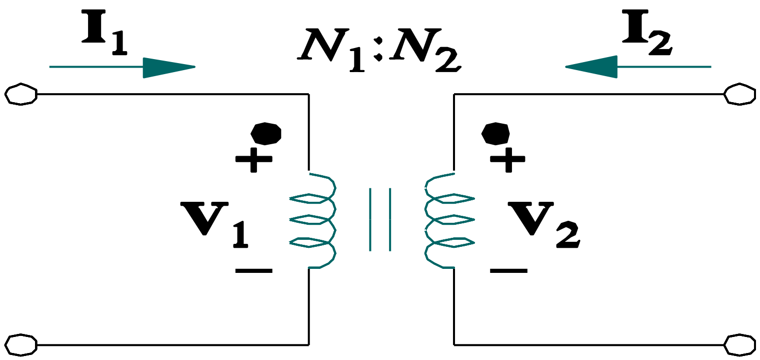


Typical circuits illustrating polarity for voltages and direction of currents of an ideal transformer



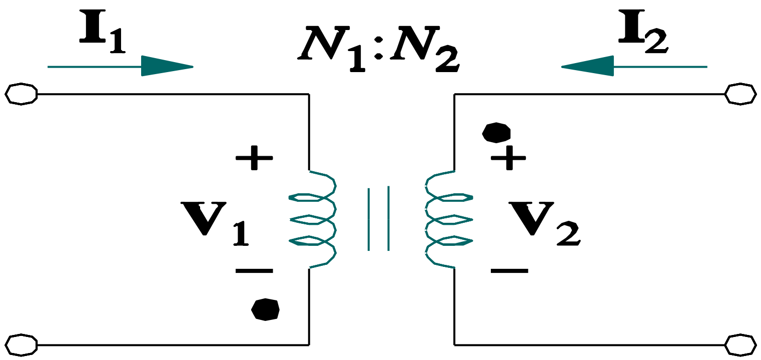
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

(a)



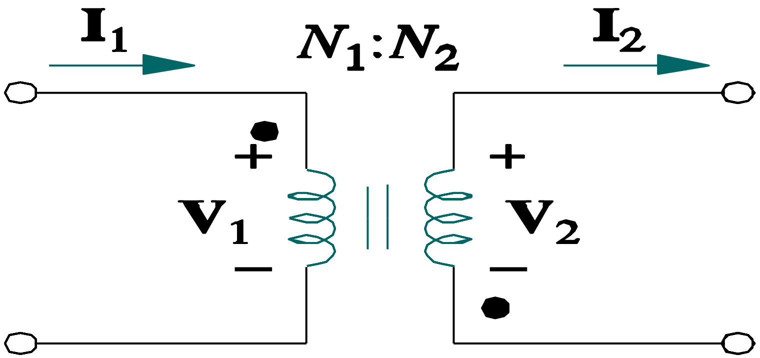
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad \frac{I_2}{I_1} = -\frac{N_1}{N_2}$$

(b)



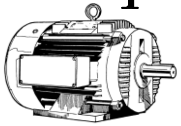
$$\frac{V_2}{V_1} = -\frac{N_2}{N_1} \qquad \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

(c)



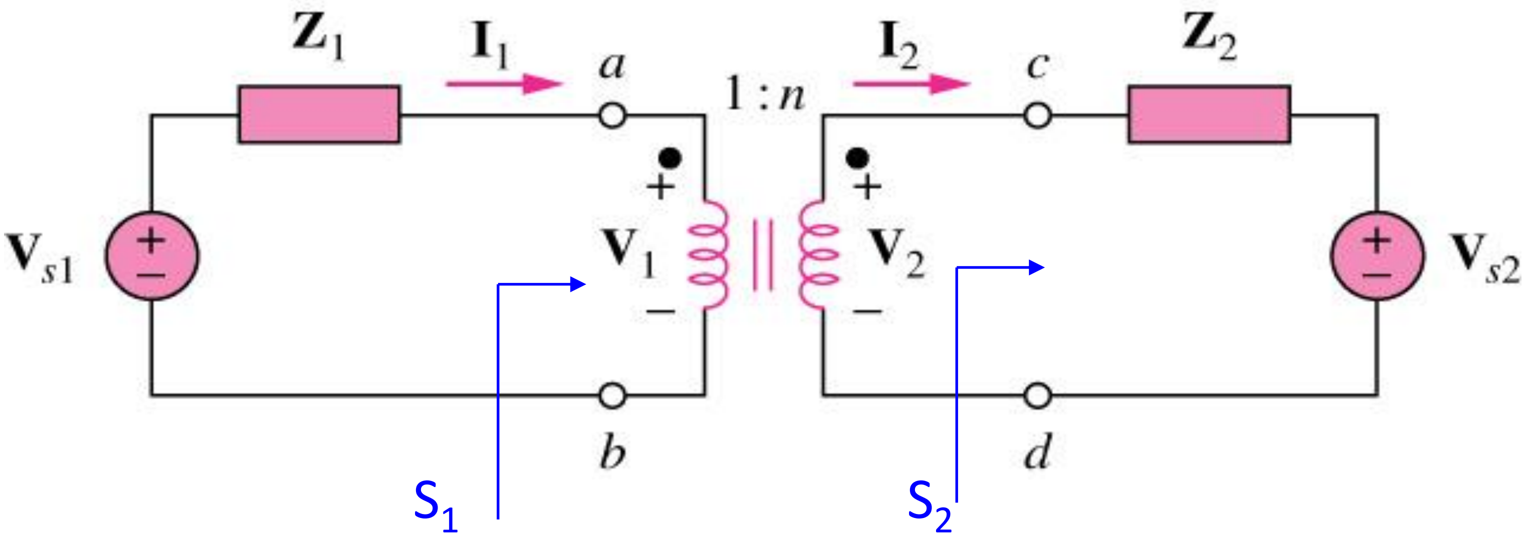
$$\frac{V_2}{V_1} = -\frac{N_2}{N_1} \qquad \frac{I_2}{I_1} = -\frac{N_1}{N_2}$$

(d)



Conservation of the Complex Power

- An ideal transformer absorbs no power.
- The complex power in the primary winding is equal to the complex power delivered to the secondary winding.
- Transformer absorbs no power. We assume a lossless transformer.



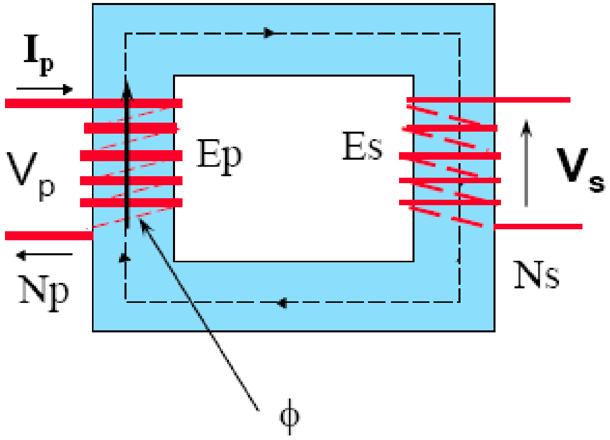
$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}^*{}_1 = \frac{\mathbf{V}_2}{n}(n\mathbf{I}^*{}_2) = \mathbf{V}_2\mathbf{I}^*{}_2 = \mathbf{S}_2$$

Ideal Transformer

- Induced Voltages: The induced emf in primary winding is:

$Ep = 4.44\ Np\ \Phi m\ f,$

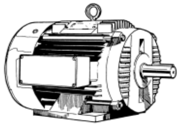
where Np is the number of winding turns in primary winding, Φm, the maximum (peak) flux, and f the frequency of the supply voltage.



- Similarly, the induced emf in secondary winding:

$Es = 4.44\ Ns\ \Phi m\ f,$

- where Ns is the number of winding turns in secondary winding.
- Turns Ratio, a = Ep/Es = Np/Ns



Ideal Transformer

❑ If the transformer is ideal,

$P_{in}=P_{out}$

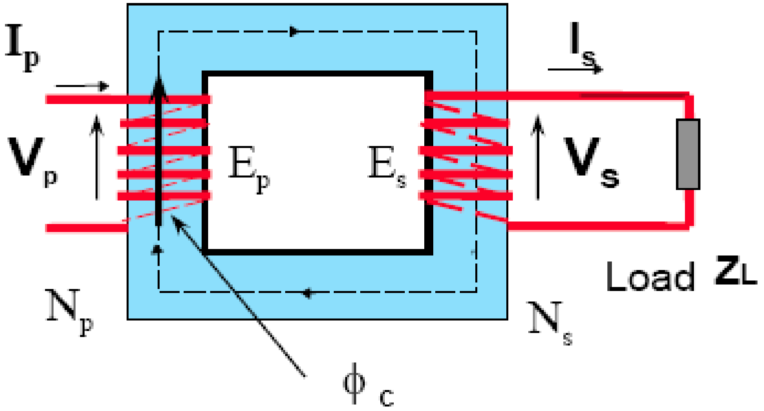
(Input power = Output power).

❑ Assuming the power factor to be same on both sides,

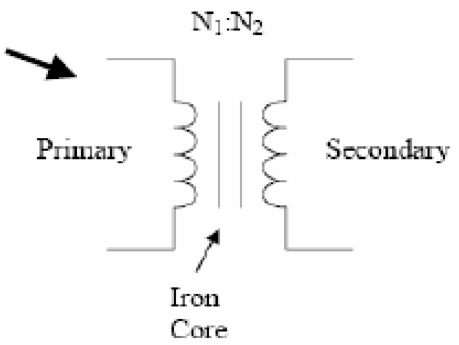
$V_p I_p = V_s I_s$

Hence, $N_p/N_s= V_p/V_s = I_s/I_p =a$

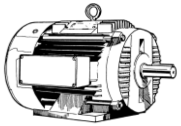
Transformer loaded



Symbol Circuit Of a transformer

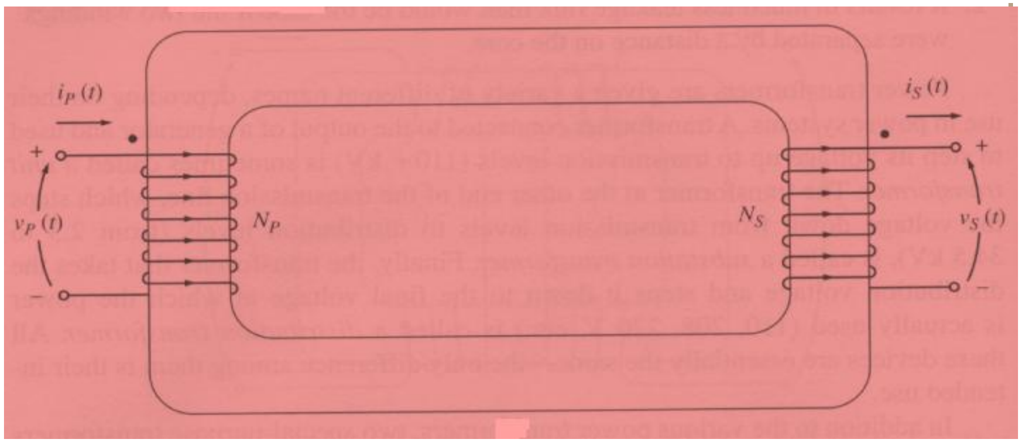


Note that in transformers, subscripts "1" and "p" are used interchangeably for the primary-side quantities. Also, subscripts "2" and "s" are used interchangeably for the secondary-side quantities.



Ideal transformer

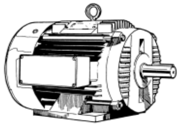
We consider a lossless transformer with an input (primary) winding having N_p turns and a secondary winding of N_s turns.



The relationship between the voltage applied to the primary winding $v_p(t)$ and the voltage produced on the secondary winding $v_s(t)$ is

$$\boxed{\frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a} \qquad (1)$$

Here a is the turn ratio of the transformer.



Ideal transformer

The relationship between the primary $i_p(t)$ and secondary $i_s(t)$ currents is

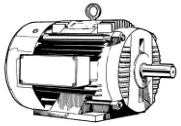
$$\boxed{\frac{i_p(t)}{i_s(t)} = \frac{1}{a}} \qquad (2)$$

In the phasor notation:

$$\boxed{\frac{\mathbf{V}_p}{\mathbf{V}_s} = a} \qquad (3)$$

$$\boxed{\frac{\mathbf{I}_p}{\mathbf{I}_s} = \frac{1}{a}} \qquad (4)$$

The phase angles of primary and secondary voltages are the same. The phase angles of primary and secondary currents are the same also. The ideal transformer changes magnitudes of voltages and currents but not their angles.



Power in an ideal transformer

Assuming that θ_p and θ_s are the angles between voltages and currents on the primary and secondary windings respectively, the power supplied to the transformer by the primary circuit is:

$$\boxed{P_{in} = V_p I_p \cos\theta_p} \tag{5}$$

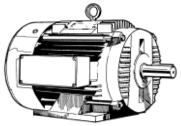
The power supplied to the output circuits is

$$\boxed{P_{out} = V_s I_s \cos\theta_s} \tag{6}$$

Since ideal transformers do not affect angles between voltages and currents:

$$\theta_p = \theta_s = \theta \tag{7}$$

Both windings of an ideal transformer have the same power factor.



Power in an ideal transformer

Since for an ideal transformer the following holds:

$$V_s = \frac{V_p}{a}; \qquad I_s = aI_p \tag{8}$$

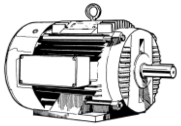
Therefore:

$$P_{out} = V_s I_s \cos\theta = \frac{V_p}{a} aI_p \cos\theta = V_p I_p \cos\theta = P_{in} \tag{9}$$

The output power of an ideal transformer equals to its input power – to be expected since assumed no loss. Similarly, for reactive and apparent powers:

$$Q_{out} = V_s I_s \sin\theta = V_p I_p \sin\theta = Q_{in} \tag{10}$$

$$S_{out} = V_s I_s = V_p I_p = S_{in} \tag{11}$$



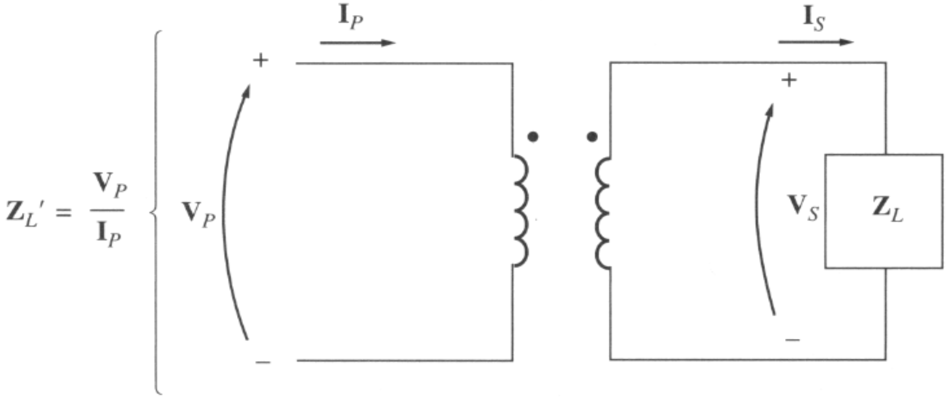
Impedance transformation

The impedance is defined as a following ratio of phasors:

$$Z_L = \mathbf{V}_L / \mathbf{I}_L \qquad (4.13.1)$$

A transformer changes voltages and currents and, therefore, an apparent impedance of the load that is given by

$$Z_L = \mathbf{V}_s / \mathbf{I}_s \qquad (12)$$

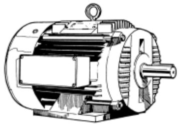


The apparent impedance of the primary circuit is:

$$Z_L' = \mathbf{V}_p / \mathbf{I}_p \qquad (13)$$

which is

$$Z_L' = \frac{\mathbf{V}_p}{\mathbf{I}_p} = \frac{a\mathbf{V}_s}{\mathbf{I}_s / a} = a^2 \frac{\mathbf{V}_s}{\mathbf{I}_s} = a^2 Z_L \qquad (14)$$



It is possible to match magnitudes of impedances (load and a transmission line) by selecting a transformer with the proper turn ratio.

Analysis of circuits containing ideal transformers

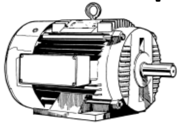
A simple method to analyze a circuit containing an ideal transformer is by replacing the portion of the circuit on one side of the transformer by an equivalent circuit with the same terminal characteristics.

Next, we exclude the transformer from the circuit and solve it for voltages and currents.

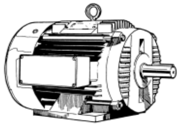
The solutions obtained for the portion of the circuit that was not replaced will be the correct values of voltages and currents of the original circuit.

Finally, the voltages and currents on the other side of the transformer (in the original circuit) can be found by considering the transformer's turn ratio.

This process is called referring of transformer's sides.

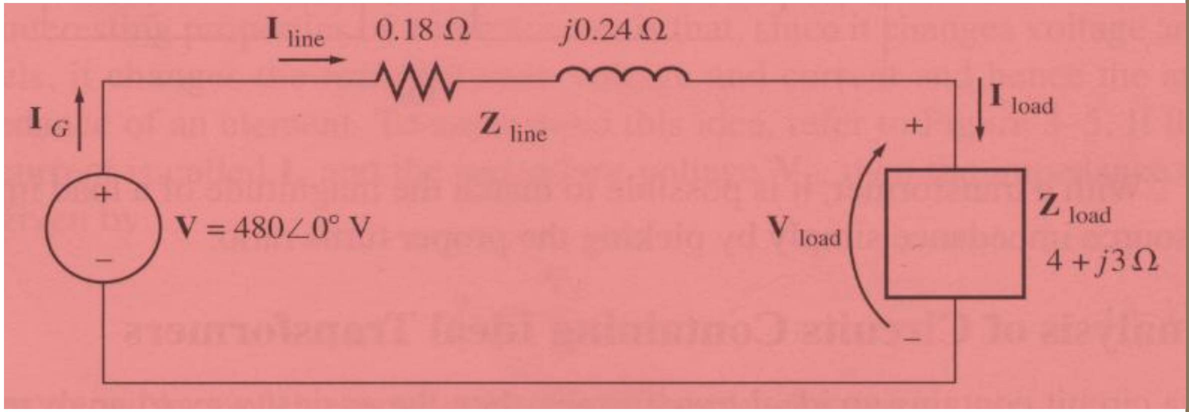


LECTURE FOUR



Analysis of circuits containing ideal transformers: Example

Example 4.1: A single-phase power system consists of a 480-V 60-Hz generator that is connected to the load Z_{load} = 4 + j3 Ω through the transmission line with Z_{line} = 0.18 + j0.24 Ω. a) What is the voltage at the load? What are the transmission line losses? b) If a 1:10 step up transformer and a 10:1 step down transformer are placed at the generator and the load ends of the transmission line respectively, what are the new load voltage and the new transmission line losses?

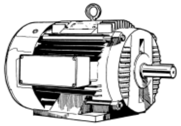


a) Here:

$$\mathbf{I}_G = \mathbf{I}_{line} = \mathbf{I}_{load} = \frac{\mathbf{V}}{\mathbf{Z}_{line} + \mathbf{Z}_{load}}$$

$$= \frac{480\angle 0°}{0.18 + j0.24 + 4 + j3}$$

$$= \frac{480\angle 0°}{5.29\angle 37.8°} = 90.8\angle -37.8°\ A$$



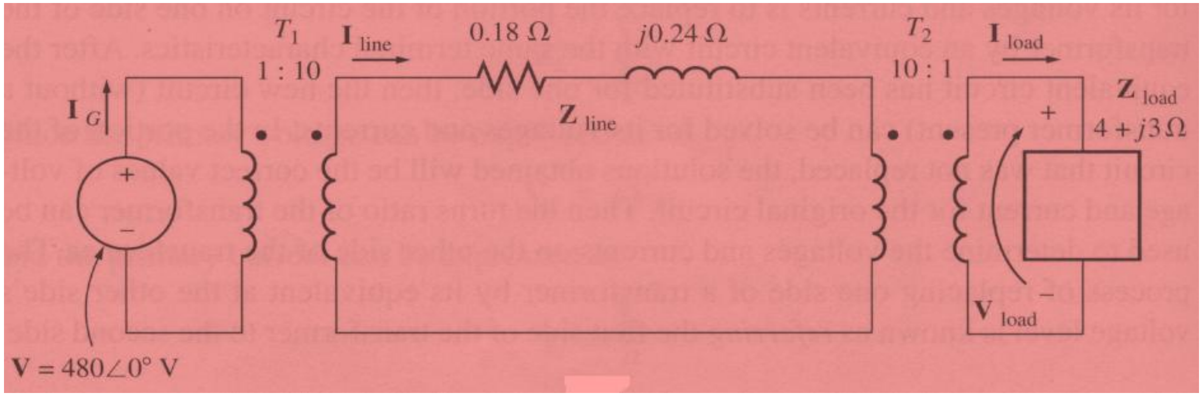
Analysis of circuits containing ideal transformers: Example

Therefore, the load voltage:

$$\mathbf{V}_{load}=\mathbf{I}_{load}\mathbf{Z}_{load}=(90.8\angle-37.8°)(4+j3)=(90.8\angle-37.8°)(5\angle36.9°)=454\angle-0.9°\,V$$

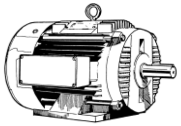
The line losses are:

$$P_{loss}=I_{line}^{2}R_{line}=90.8^{2}\cdot0.18=1484\ W$$



b) We will

1) eliminate transformer T_2 by referring the load over to the transmission line's voltage level.
2) Eliminate transformer T_1 by referring the transmission line's elements and the equivalent load at the transmission line's voltage over to the source side.



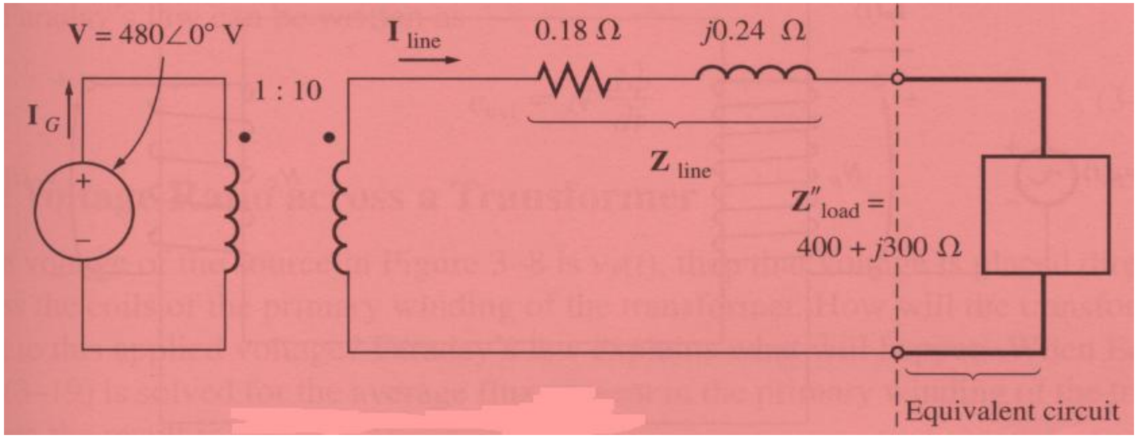
Analysis of circuits containing ideal transformers: Example

The load impedance when referred to the transmission line (while the transformer T_2 is eliminated) is:

$$\mathbf{Z}'_{load} = a_2^2 \mathbf{Z}_{load} = \left(\frac{10}{1}\right)^2 (4 + j3) = 400 + j300$$

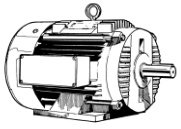
The total impedance on the transmission line level is

$$\mathbf{Z}_{eq} = \mathbf{Z}_{line} + \mathbf{Z}'_{load}$$
$$= 400.18 + j300.24$$
$$= 500.3\angle 36.88^\circ \ \Omega$$



The total impedance is now referred across T_1 to the source's voltage level:

$$\mathbf{Z}'_{eq} = a_1^2 \mathbf{Z}_{eq} = \left(\frac{1}{10}\right)^2 (500.3\angle 36.88^\circ) = 5.003\angle 36.88^\circ \ \Omega$$

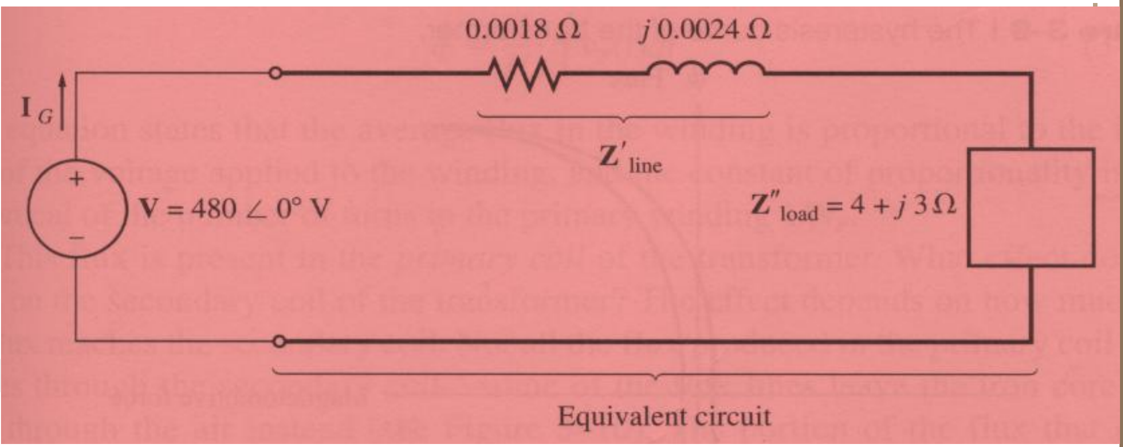


Analysis of circuits containing ideal transformers: Example

The generator's current is

$$\mathbf{I}_G = \frac{\mathbf{V}}{\mathbf{Z}'_{eq}} = \frac{480\angle 0^\circ}{5.003\angle 36.88^\circ}$$

$$= 95.94\angle -36.88^\circ \quad A$$



Knowing transformers' turn ratios, we can determine line and load currents:

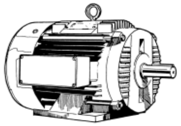
$$\mathbf{I}_{line} = a_1 \mathbf{I}_G = 0.1 \cdot \left(95.94\angle -36.88^\circ\right) = 9.594\angle -36.88^\circ \quad A$$

$$\mathbf{I}_{load} = a_2 \mathbf{I}_{line} = 10 \cdot \left(9.594\angle -36.88^\circ\right) = 95.94\angle -36.88^\circ \quad A$$

Therefore, the load voltage is:

$$\mathbf{V}_{load} = \mathbf{I}_{load} \mathbf{Z}_{load} = \left(95.94\angle -36.88^\circ\right)\left(5\angle -36.87^\circ\right) = 479.7\angle -0.01^\circ \quad V$$

The losses in the line are: $P_{loss} = I_{line}^2 R_{line} = 9.594^2 \cdot 0.18 = 16.7 \quad W$



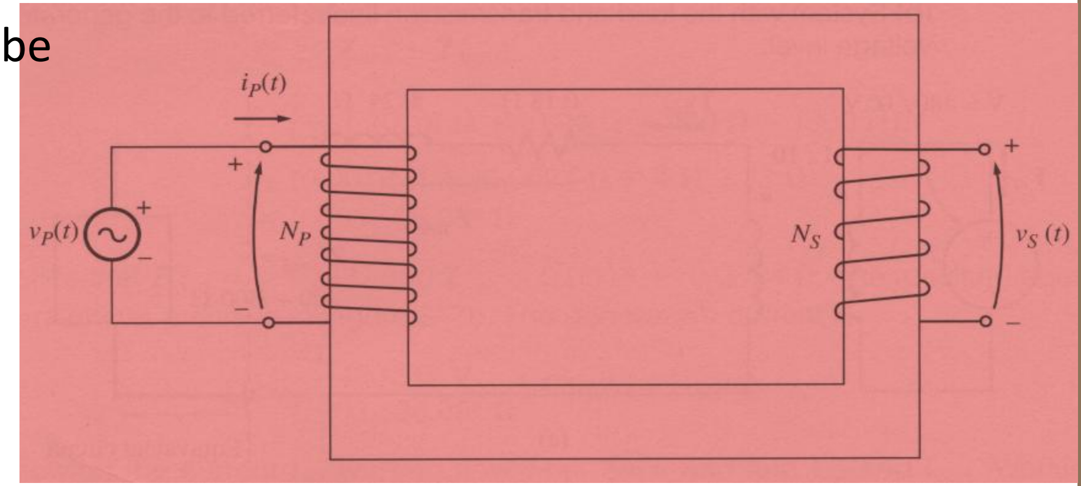
Note: transmission line losses are reduced by a factor nearly 90, the load voltage is much closer to the generator's voltage – effects of increasing the line's voltage.

Theory of operation of real single-phase transformers

Real transformers approximate ideal ones to some degree.

The basis transformer operation can be derived from Faraday's law:

$$e_{ind} = \frac{d\lambda}{dt} \qquad (15)$$



Here λ is the flux linkage in the coil across which the voltage is induced:

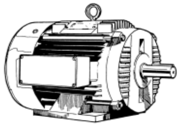
$$\lambda = \sum_{i=1}^{N} \phi_i \qquad (16)$$

where ϕ_i is the flux passing through the i^{th} turn in a coil – slightly different for different turns. However, we may use an average flux per turn in the coil having N turns:

$$\bar{\phi} = \lambda / N \qquad (17)$$

Therefore:

$$e_{ind} = N\frac{d\bar{\phi}}{dt} \qquad (18)$$



The voltage ratio across a real transformer

If the source voltage $v_p(t)$ is applied to the primary winding, the average flux in the primary winding will be:

$$\bar{\phi} = \frac{1}{N_p}\int v_p(t)\,dt \qquad (19)$$

A portion of the flux produced in the primary coil passes through the secondary coil (mutual flux); the rest is lost (leakage flux):

$$\bar{\phi}_p = \phi_m + \phi_{Lp} \qquad (20)$$

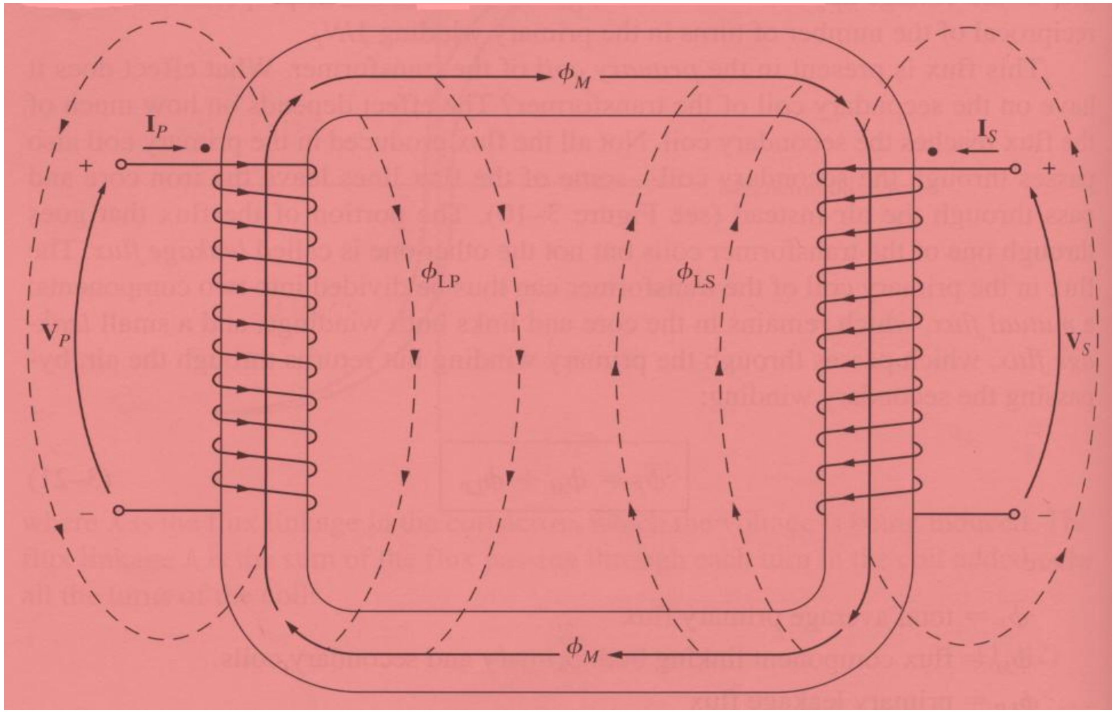
average primary flux

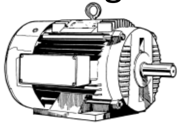
mutual flux

Similarly, for the secondary coil:

$$\bar{\phi}_s = \phi_m + \phi_{Ls} \qquad (21)$$

Average secondary flux





The voltage ratio across a real transformer

From the Faraday's law, the primary coil's voltage is:

$$v_p(t) = N_p \frac{d\bar{\phi}_p}{dt} = N_p \frac{d\phi_m}{dt} + N_p \frac{d\phi_{Lp}}{dt} = e_p(t) + e_{Lp}(t) \qquad (22)$$

The secondary coil's voltage is:

$$v_s(t) = N_s \frac{d\bar{\phi}_s}{dt} = N_s \frac{d\phi_m}{dt} + N_s \frac{d\phi_{Ls}}{dt} = e_s(t) + e_{Ls}(t) \qquad (23)$$

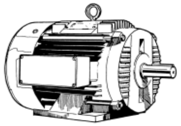
The primary and secondary voltages due to the mutual flux are:

$$e_p(t) = N_p \frac{d\phi_m}{dt} \qquad (23)$$

$$e_s(t) = N_s \frac{d\phi_{Ls}}{dt} \qquad (24)$$

Combining the last two equations:

$$\frac{e_p(t)}{N_p} = \frac{d\phi_m}{dt} = \frac{e_s(t)}{N_s} \qquad (25)$$



The voltage ratio across a real transformer

Therefore:

$$\boxed{\frac{e_p(t)}{e_s(t)} = \frac{N_p}{N_s} = a}$$ (26)

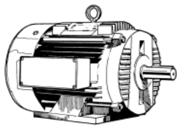
That is, the ratio of the primary voltage to the secondary voltage both caused by the mutual flux is equal to the turns ratio of the transformer.

For well-designed transformers:

$$\phi_m \;\square\; \phi_{Lp}; \qquad \phi_m \;\square\; \phi_{Ls}$$ (27)

Therefore, the following approximation normally holds:

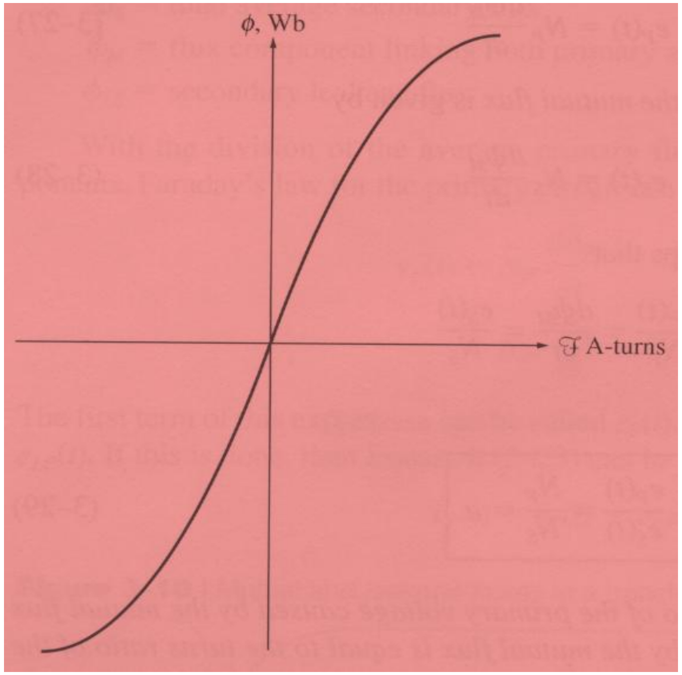
$$\frac{v_p(t)}{v_s(t)} \approx \frac{N_p}{N_s} \approx a$$ (28)



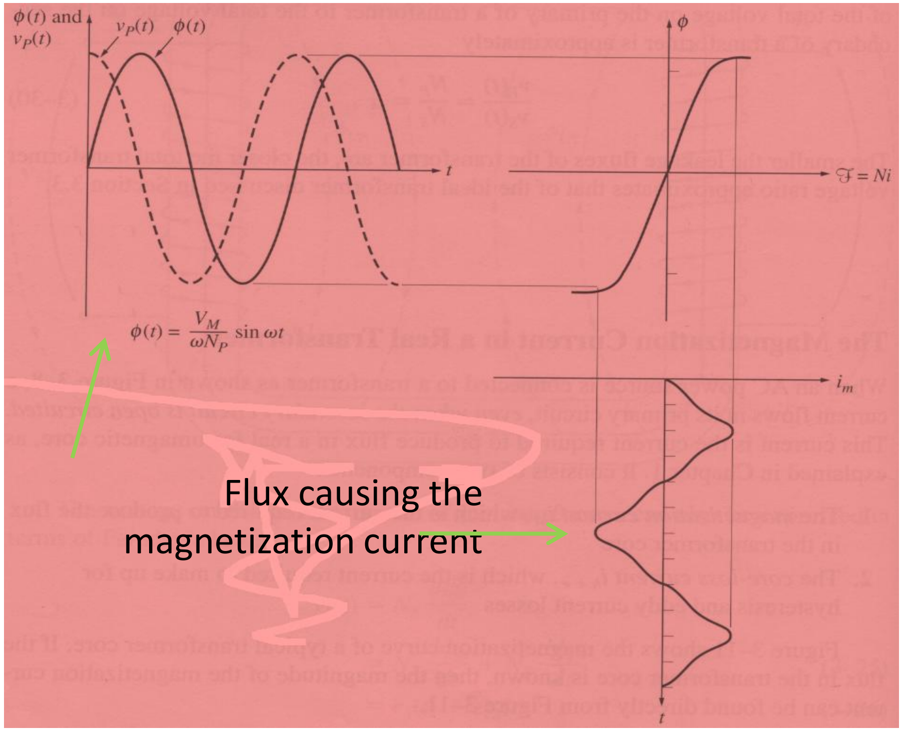
The magnetization current in a real transformer

Even when no load is connected to the secondary coil of the transformer, a current will flow in the primary coil. This current consists of:

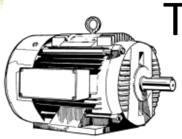
1. The magnetization current i_m needed to produce the flux in the core;
2. The core-loss current i_{h+e} hysteresis and eddy current losses.



Typical magnetization curve



Flux causing the magnetization current



The magnetization current in a real transformer

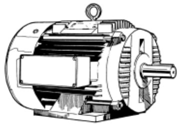
Ignoring flux leakage and assuming time-harmonic primary voltage, the average flux is:

$$\bar{\phi} = \frac{1}{N_p}\int v_p(t)dt = \frac{1}{N_p}\int V_m \cos\omega t dt = \frac{V_m}{\omega N_p}\sin\omega t \quad [Wb] \qquad (29)$$

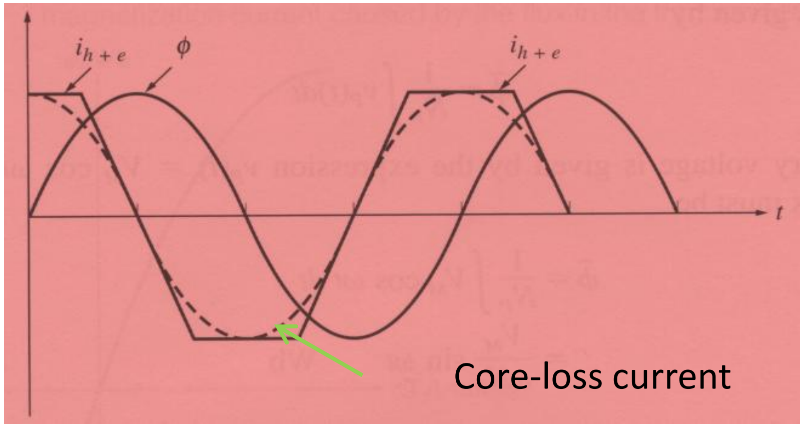
If the values of current are comparable to the flux they produce in the core, it is possible to sketch a magnetization current. We observe:

1. Magnetization current is not sinusoidal: there are high frequency components;
2. Once saturation is reached, a small increase in flux requires a large increase in magnetization current;
3. Magnetization current (its fundamental component) lags the voltage by 90°;
4. High-frequency components of the current may be large in saturation.

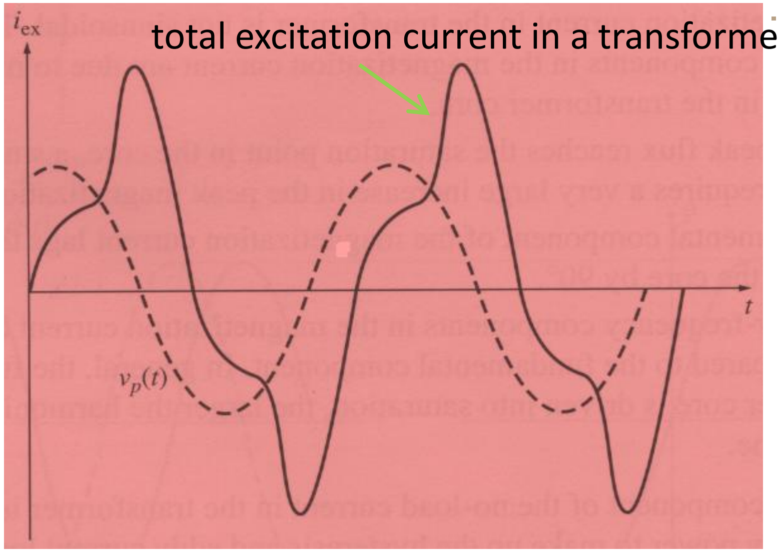
Assuming a sinusoidal flux in the core, the eddy currents will be largest when flux passes zero.



The magnetization current in a real transformer



Core-loss current



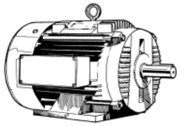
total excitation current in a transformer

Core-loss current is:

1. Nonlinear due to nonlinear effects of hysteresis;
2. In phase with the voltage.

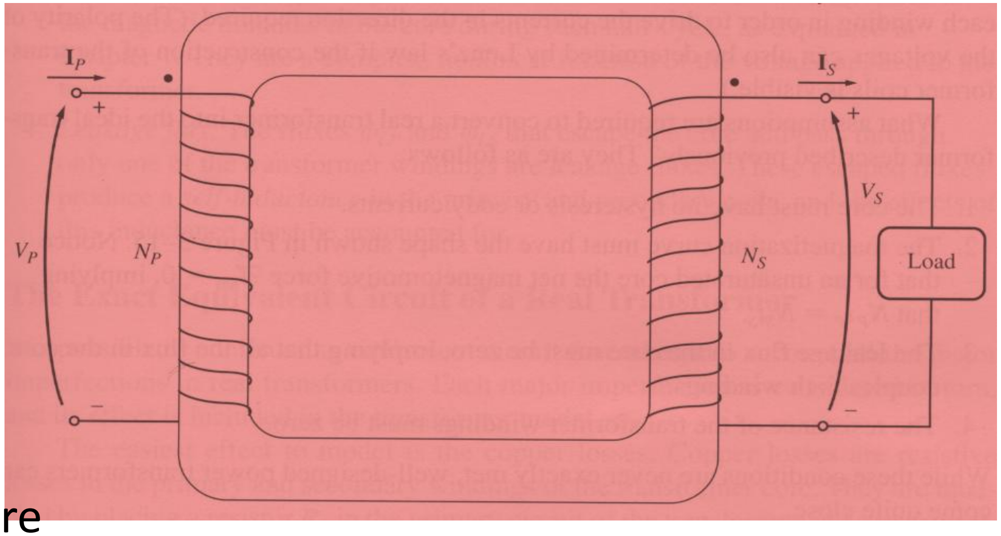
The total no-load current in the core is called the excitation current of the transformer:

$$i_{ex} = i_m + i_{h+e} \qquad (30)$$



The current ratio on a transformer

If a load is connected to the secondary coil, there will be a current flowing through it.



A current flowing into the dotted end of a winding produces a positive magnetomotive force *F*:

$$F_p = N_p i_p \qquad (31)$$

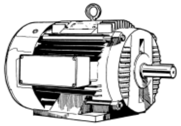
$$F_s = N_s i_s \qquad (32)$$

The net magnetomotive force in the core

$$F_{net} = N_p i_p - N_s i_s = \phi \Re \qquad (33)$$

where $\Re$ is the reluctance of the transformer core. For well-designed transformer cores, the reluctance is very small if the core is not saturated. Therefore:

$$F_{net} = N_p i_p - N_s i_s \approx 0 \qquad (34)$$



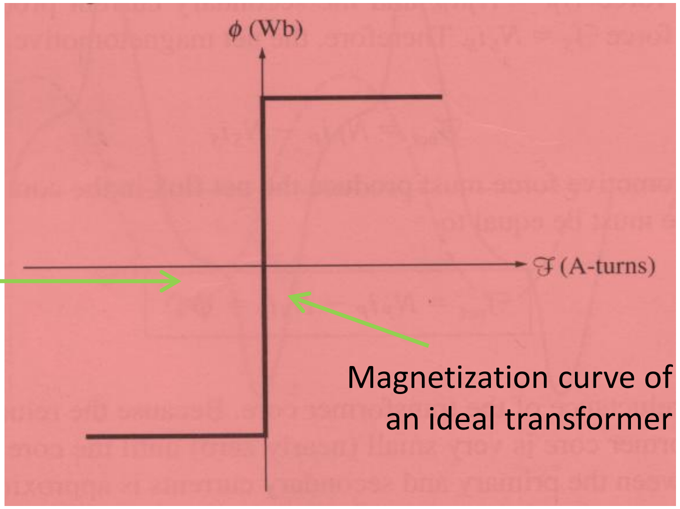
The current ratio on a transformer

The last approximation is valid for well-designed unsaturated cores. Therefore:

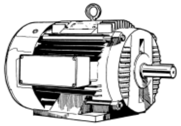
$$N_p i_p \approx N_s i_s \quad \Rightarrow \quad \frac{i_p}{i_s} \approx \frac{N_s}{N_p} = \frac{1}{a} \qquad (35)$$

An ideal transformer (unlike the real one) can be characterized as follows:

1. The core has no hysteresis or eddy currents.
2. The magnetization curve is
3. The leakage flux in the core is zero.
4. The resistance of the windings is zero.



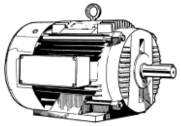
Magnetization curve of an ideal transformer



The transformer’s equivalent circuit

To model a real transformer accurately, we need to account for the following losses:

1. Copper losses – resistive heating in the windings: I^2R.
2. Eddy current losses – resistive heating in the core: proportional to the square of voltage applied to the transformer.
3. Hysteresis losses – energy needed to rearrange magnetic domains in the core: nonlinear function of the voltage applied to the transformer.
4. Leakage flux – flux that escapes from the core and flux that passes through one winding only.

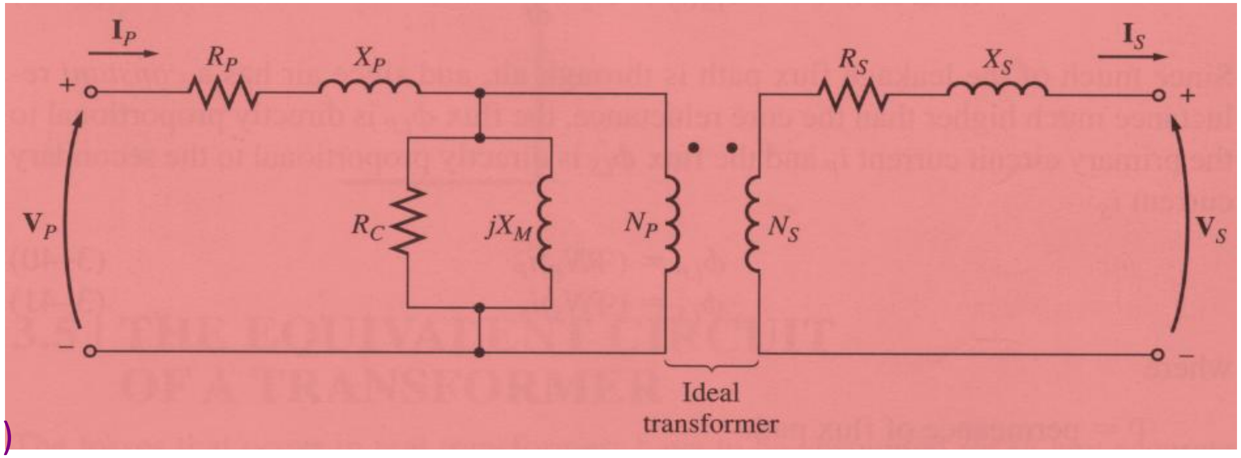


The exact equivalent circuit of a real transformer

Cooper losses are modeled by the resistors R_p and R_s.

Leakage flux in a primary winding produces the voltage:

$$e_{Lp}(t) = N_p \frac{d\phi_{Lp}}{dt} \qquad (36)$$



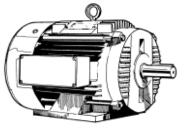
Since much of the leakage flux pass through air, and air has a constant reluctance that is much higher than the core reluctance, the primary coil's leakage flux is:

$$\phi_{Lp} = \underset{\uparrow}{\mathcal{P}} N_p i_p \qquad (37)$$

permeance of flux path

Therefore:

$$e_{Lp}(t) = N_p \frac{d}{dt}\left(\mathcal{P} N_p i_p\right) = N_p^2 \mathcal{P} \frac{di_p}{dt} \qquad (38)$$



The exact equivalent circuit of a real transformer

Recognizing that the self-inductance of the primary coil is

$$L_p = N_p^2 \mathcal{P} \qquad (39)$$

The induced voltages are:

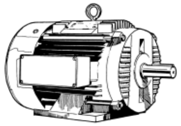
Primary coil:

$$\boxed{e_{Lp}(t) = L_p \frac{di_p}{dt}} \qquad (40)$$

Secondary coil:

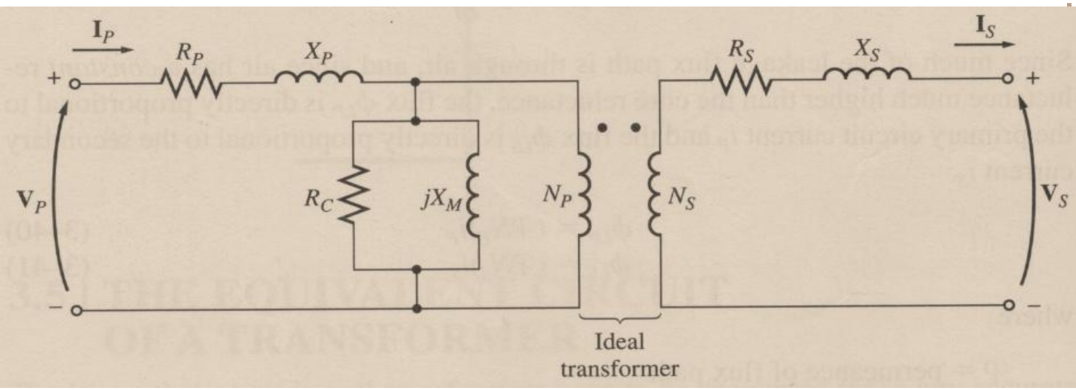
$$\boxed{e_{Ls}(t) = L_s \frac{di_s}{dt}} \qquad (41)$$

The leakage flux can be modeled by primary and secondary inductors.
The magnetization current can be modeled by a reactance X_M connected across the primary voltage source.
The core-loss current can be modeled by a resistance R_C connected across the primary voltage source.
Both currents are nonlinear; therefore, X_M and R_C are just approximations.



The exact equivalent circuit of a real transformer

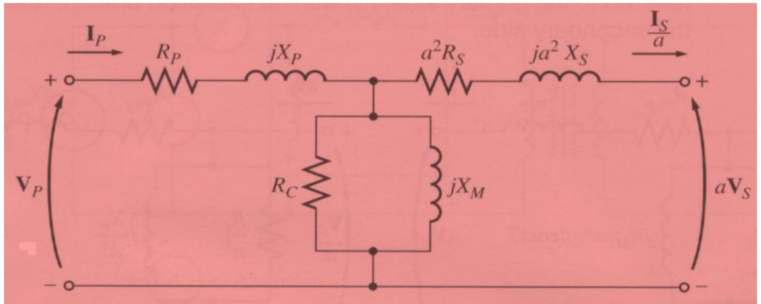
The transformer’s equivalent circuit



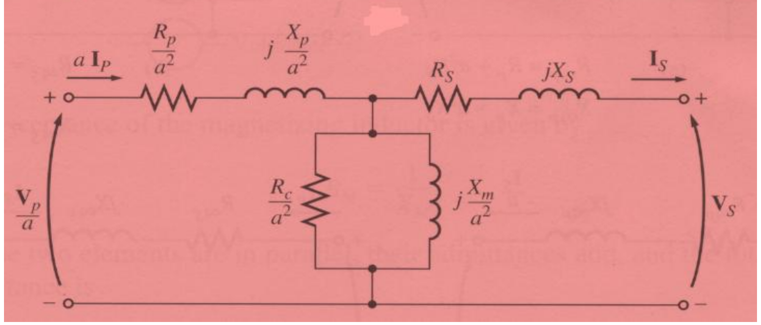
However, the exact circuit is not very practical.

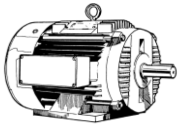
Therefore, the equivalent circuit is usually referred to the primary side or the secondary side of the transformer.

Equivalent circuit of the transformer referred to its primary side.



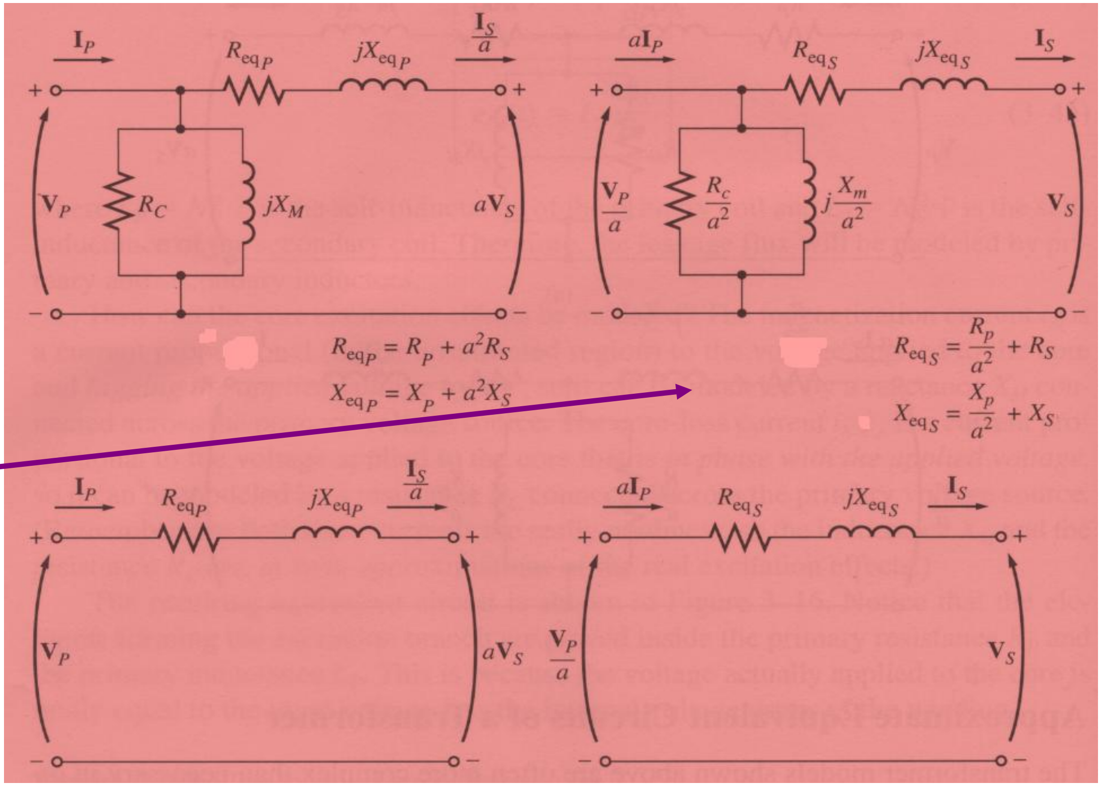
Equivalent circuit of the transformer referred to its secondary side.





Approximate equivalent circuit of a transformer

For many practical applications, approximate models of transformers are used.



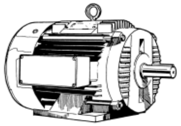
Referred to the primary side.

Referred to the secondary side.

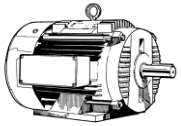
Without an excitation branch referred to the primary side.

The values of components of the transformer model can be determined experimentally by an open-circuit test or by a short-circuit test.

Without an excitation branch referred to the secondary side.



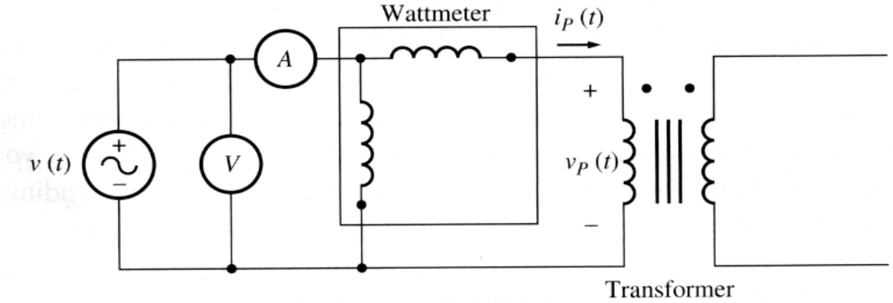
LECTURE FIVE



Determining the values of components

The open-circuit test.

Full line voltage is applied to the primary side of the transformer. The input voltage, current, and power are measured.



From this information, the power factor of the input current and the magnitude and the angle of the excitation impedance can be determined.

To evaluate R_C and X_M, we determine the conductance of the core-loss resistor is:

$$G_C = \frac{1}{R_C} \tag{42}$$

The susceptance of the magnetizing inductor is:

$$B_M = \frac{1}{X_M} \tag{43}$$

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Determining the values of components

Since both elements are in parallel, their admittances add. Therefore, the total excitation admittance is:

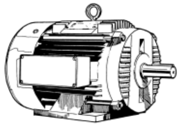
$$Y_E = G_C - jB_M = \frac{1}{R_C} - j\frac{1}{X_M} \tag{44}$$

The magnitude of the excitation admittance in the open-circuit test is:

$$\left|Y_E\right| = \frac{I_{oc}}{V_{oc}} \tag{45}$$

The angle of the admittance in the open-circuit test can be found from the circuit power factor (PF):

$$\cos\theta = PF = \frac{P_{oc}}{V_{oc} I_{oc}} \tag{46}$$

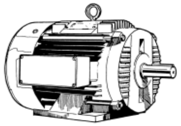


Determining the values of components

In real transformers, the power factor is always lagging, so the angle of the current always lags the angle of the voltage by θ degrees. The admittance is:

$$Y_E = \frac{I_{oc}}{V_{oc}} \angle -\theta = \frac{I_{oc}}{V_{oc}} \angle -\cos^{-1} PF \qquad (4.35.1)$$

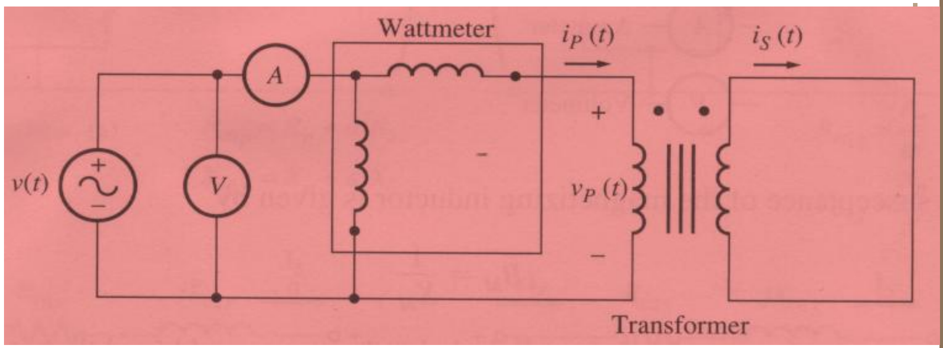
Therefore, it is possible to determine values of R_C and X_M in the open-circuit test.



Determining the values of components

The short-circuit test.

Fairly low input voltage is applied to the primary side of the transformer. This voltage is adjusted until the current in the secondary winding equals to its rated value.



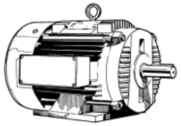
The input voltage, current, and power are again measured.

Since the input voltage is low, the current flowing through the excitation branch is negligible; therefore, all the voltage drop in the transformer is due to the series elements in the circuit. The magnitude of the series impedance referred to the primary side of the transformer is:

$$\left|Z_{SE}\right| = \frac{V_{SC}}{I_{SC}} \tag{47}$$

The power factor of the current is given by:

$$PF = \cos\theta = \frac{P_{SC}}{V_{SC} I_{SC}} \tag{48}$$



Determining the values of components

Therefore:

$$Z_{SE}=\frac{V_{SC}\angle 0^\circ}{I_{SC}\angle -\theta^\circ}=\frac{V_{SC}}{I_{SC}}\angle\theta^\circ \qquad (49)$$

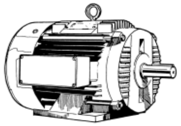
Since the serial impedance Z_{SE} is equal to

$$Z_{SE}=R_{eq}+jX_{eq} \qquad (50)$$

$$Z_{SE}=\left(R_p+a^2R_S\right)+j\left(X_p+a^2X_S\right) \qquad (51)$$

it is possible to determine the total series impedance referred to the primary side of the transformer. However, there is no easy way to split the series impedance into primary and secondary components.

The same tests can be performed on the secondary side of the transformer. The results will yield the equivalent circuit impedances referred to the secondary side of the transformer.



Determining the values of components: Example

Example 4.2: We need to determine the equivalent circuit impedances of a 20 kVA, 8000/240 V, 60 Hz transformer. The open-circuit and short-circuit tests led to the following data:

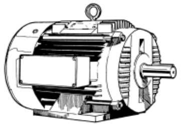
$V_{OC} = 8000\ V$	$V_{SC} = 489\ V$
$I_{OC} = 0.214\ A$	$I_{SC} = 2.5\ A$
$P_{OC} = 400\ W$	$P_{SC} = 240\ W$

The power factor during the open-circuit test is

$$PF = \cos\theta = \frac{P_{OC}}{V_{OC} I_{OC}} = \frac{400}{8000 \cdot 0.214} = 0.234 \;\; lagging$$

The excitation admittance is

$$Y_E = \frac{I_{oc}}{V_{oc}} \angle -\cos^{-1} PF = \frac{0.214}{8000} \angle -\cos^{-1} 0.234 = 0.0000063 - j0.0000261 = \frac{1}{R_C} - j\frac{1}{X_M}$$



Determining the values of components: Example

Therefore:

$$R_C = \frac{1}{0.0000063} = 159\,k\Omega; \quad X_M = \frac{1}{0.0000261} = 38.3\,k\Omega$$

The power factor during the short-circuit test is

$$PF = \cos\theta = \frac{P_{SC}}{V_{SC} I_{SC}} = \frac{240}{489 \cdot 2.5} = 0.196 \;\; lagging$$

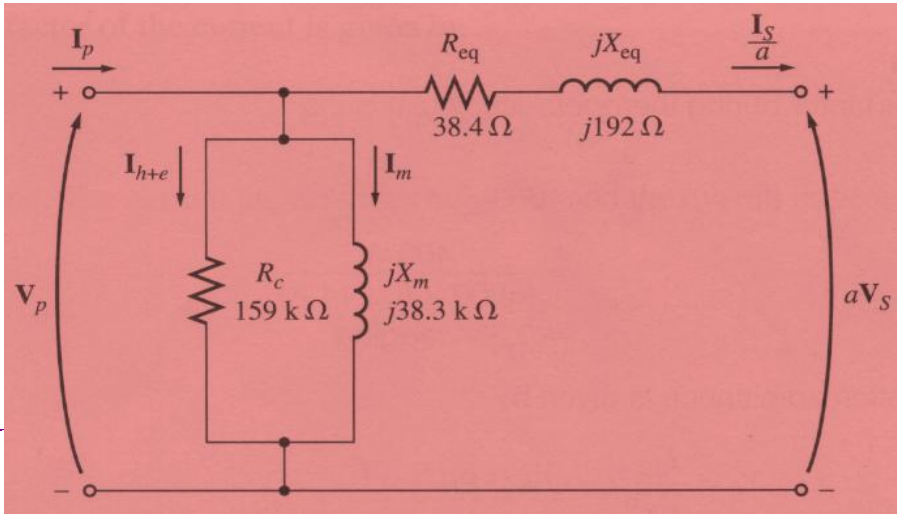
The series impedance is given by

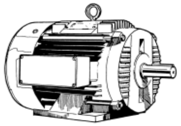
$$Z_{SE} = \frac{V_{SC}}{I_{SC}} \angle \cos^{-1} PF = \frac{489}{2.5} \angle 78.7°$$

$$= 38.4 + j192\,\Omega$$

Therefore:

$$R_{eq} = 38.3\,\Omega; \quad X_{eq} = 192\,\Omega$$





The equivalent circuit

The per-unit system

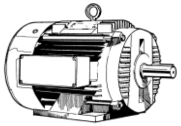
Another approach to solve circuits containing transformers is the per-unit system. Impedance and voltage-level conversions are avoided. Also, machine and transformer impedances fall within fairly narrow ranges for each type and construction of device while the per-unit system is employed.

$$Quantity\ per\ unit = \frac{actual\ value}{base\ value\ of\ quantity} \tag{52}$$

Usually, two base quantities are selected to define a given per-unit system. Often, such quantities are voltage and power (or apparent power). In a 1-phase system:

$$P_{base},\, Q_{base},\ or\ \ S_{base} = V_{base} I_{base} \tag{53}$$

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{\left(V_{base}\right)^2}{S_{base}} \tag{54}$$



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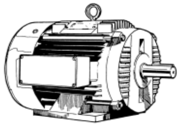
The per-unit system

$$Y_{base} = \frac{I_{base}}{V_{base}} \qquad (4.41.1)$$

Ones the base values of *P* (or *S*) and *V* are selected, all other base values can be computed form the above equations.

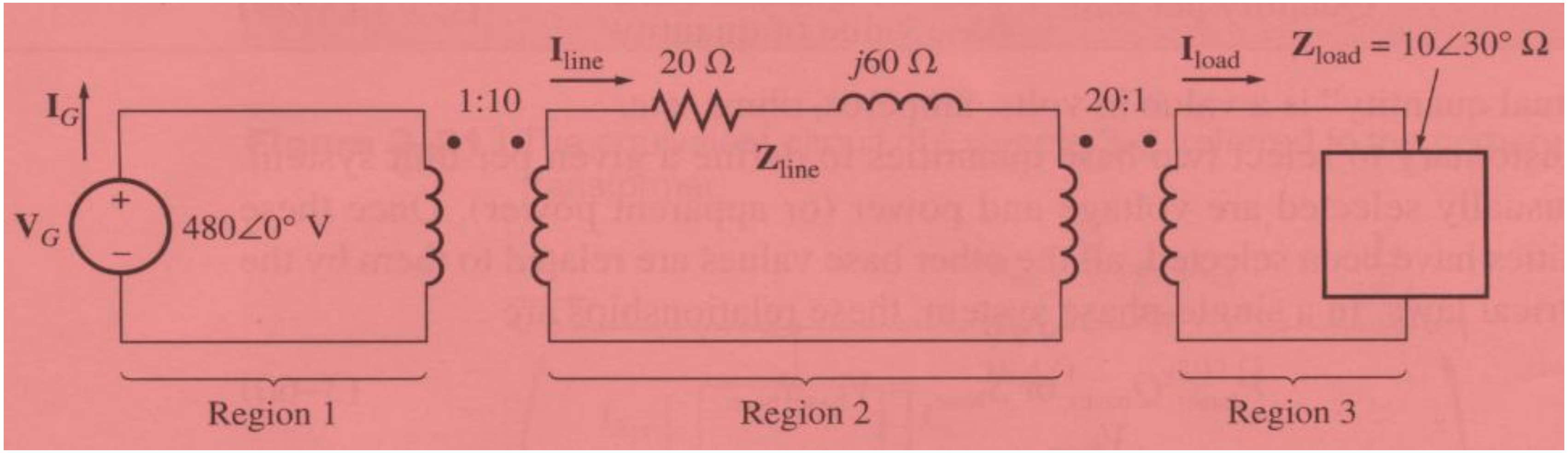
In a power system, a base apparent power and voltage are selected at the specific point in the system. Note that a transformer has no effect on the apparent power of the system, since the apparent power into a transformer equals the apparent power out of a transformer. As a result, the base apparent power remains constant everywhere in the power system.

On the other hand, voltage (and, therefore, a base voltage) changes when it goes through a transformer according to its turn ratio. Therefore, the process of referring quantities to a common voltage level is done automatically in the per-unit system.



The per-unit system: Example

Example 4.3: A simple power system is given by the circuit:



The generator is rated at 480 V and 10 kVA.

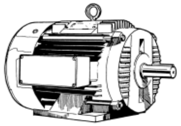
a) Find the base voltage, current, impedance, and apparent power at every points in the power system;

b) Convert the system to its per-unit equivalent circuit;

c) Find the power supplied to the load in this system;

e) Find the power lost in the transmission line (Region 2).

LECTURE FOUR



The per-unit system: Example

a. In the generator region: $V_{base\ 1} = 480\ V$ and $S_{base} = 10\ kVA$

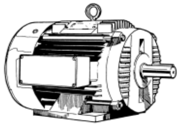
$$I_{base\,1} = \frac{S_{base\,1}}{V_{base\,1}} = \frac{10\,000}{480} = 20.83\ A$$

$$Z_{base\,1} = \frac{V_{base\,1}}{I_{base\,1}} = \frac{480}{20.83} = 23.04\,\Omega$$

The turns ratio of the transformer T_1 is $a_1 = 0.1$; therefore, the voltage in the transmission line region is

$$V_{base\,2} = \frac{V_{base\,1}}{a_1} = \frac{480}{0.1} = 4800\ V$$

The other base quantities are



The per-unit system: Example

$$S_{base\,2} = 10 \;\; kVA$$

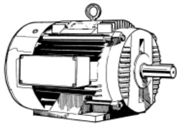
$$I_{base\,2} = \frac{10\,000}{4800} = 2.083 \; A$$

$$Z_{base\,2} = \frac{4800}{2.083} = 2304 \; \Omega$$

The turns ratio of the transformer T_2 is $a_2 = 20$; therefore, the voltage in the load region is

$$V_{base\,3} = \frac{V_{base\,2}}{a_2} = \frac{4800}{20} = 240 \; V$$

The other base quantities are



The per-unit system: Example

$$S_{base\,3} = 10\ \ kVA$$

$$I_{base\,3} = \frac{10\,000}{240} = 41.67\ A$$

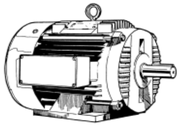
$$Z_{base\,3} = \frac{240}{41.67} = 5.76\,\Omega$$

b. To convert a power system to a per-unit system, each component must be divided by its base value in its region. The generator's per-unit voltage is

$$V_{G,pu} = \frac{480\angle 0^\circ}{480} = 1.0\angle 0^\circ\ pu$$

The transmission line's per-unit impedance is

$$Z_{line,\,pu} = \frac{20 + j60}{2304} = 0.0087 + j0.026\ pu$$

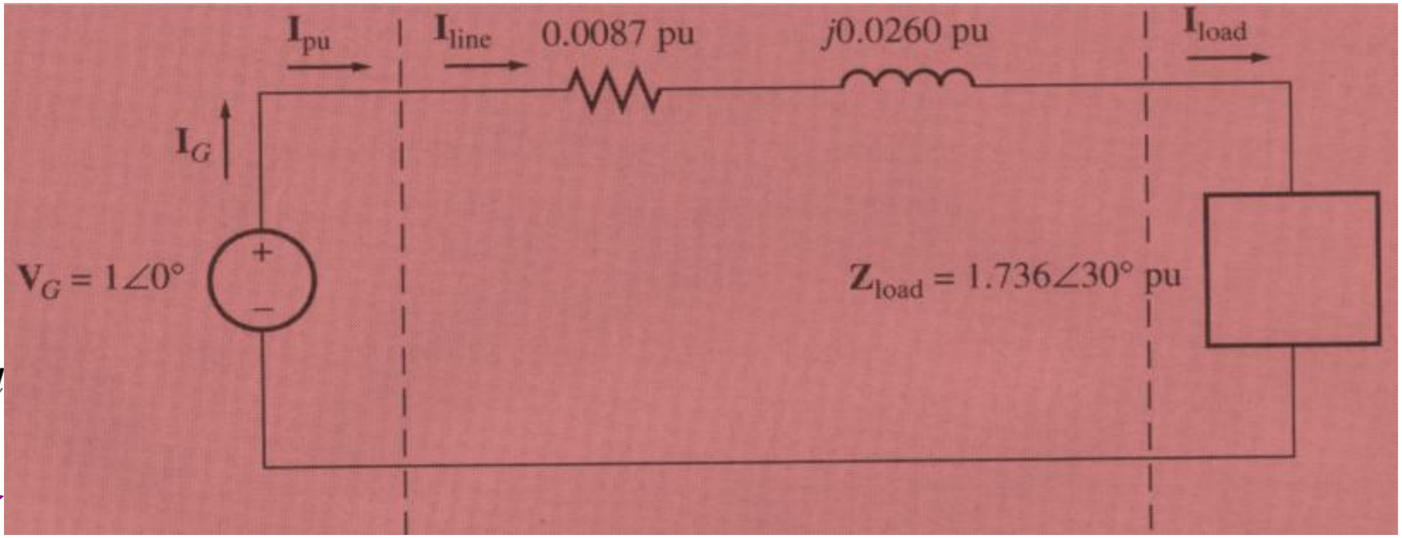


The per-unit system: Example

The load's per-unit impedance is

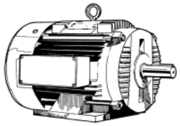
$$Z_{load,\,pu} = \frac{10\angle 30^\circ}{5.76}$$
$$= 1.736\angle 30^\circ \; pu$$

The per-unit equivalent circuit



c. The current flowing in this per-unit power system is

$$I_{pu} = \frac{V_{pu}}{Z_{tot,pu}} = \frac{1\angle 0^\circ}{0.0087 + j0.026 + 1.736\angle 30^\circ} = 0.569\angle -30.6^\circ \; pu$$



The per-unit system: Example

Therefore, the per-unit power on the load is

$$P_{load,\,pu} = I_{pu}^2 R_{pu} = 0.569^2 \cdot 1.503 = 0.487$$

The actual power on the load is

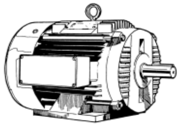
$$P_{load} = P_{load,pu} S_{base} = 0.487 \cdot 10\,000 = 487\,W$$

d. The per-unit power lost in the transmission line is

$$P_{line,pu} = I_{pu}^2 R_{line,\,pu} = 0.569^2 \cdot 0.0087 = 0.00282$$

The actual power lost in the transmission line

$$P_{line} = P_{line,pu} S_{base} = 0.00282 \cdot 10\,000 = 28.2\,W$$

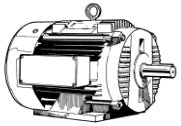


The per-unit system

When only one device (transformer or motor) is analyzed, its own ratings are used as the basis for per-unit system. When considering a transformer in a per-unit system, transformer's characteristics will not vary much over a wide range of voltages and powers. For example, the series resistance is usually from 0.02 to 0.1 pu; the magnetizing reactance is usually from 10 to 40 pu; the core-loss resistance is usually from 50 to 200 pu. Also, the per-unit impedances of synchronous and induction machines fall within relatively narrow ranges over quite large size ranges.

If more than one transformer is present in a system, the system base voltage and power can be chosen arbitrary. However, the entire system must have the same base power, and the base voltages at various points in the system must be related by the voltage ratios of the transformers.

System base quantities are commonly chosen to the base of the largest component in the system.



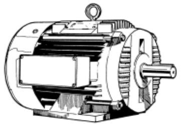
The per-unit system

Per-unit values given to another base can be converted to the new base either through an intermediate step (converting them to the actual values) or directly as follows:

$$\left(P,Q,S\right)_{pu,\,base\,2}=\left(P,Q,S\right)_{pu,\,base\,1}\frac{S_{base\,1}}{S_{base\,2}} \qquad (4.49.1)$$

$$V_{pu,\,base\,2}=V_{pu,\,base\,1}\frac{V_{base\,1}}{V_{base\,2}} \qquad (4.49.2)$$

$$\left(R,X,Z\right)_{pu,\,base\,2}=\left(R,X,Z\right)_{pu,\,base\,1}\frac{V_{base\,1}^{2}S_{base\,1}}{V_{base\,2}^{2}S_{base\,2}} \qquad (4.49.3)$$



The per-unit system: Example

Example 4.4: Sketch the appropriate per-unit equivalent circuit for the 8000/240 V, 60 Hz, 20 kVA transformer with R_c = 159 kΩ, X_M = 38.4 kΩ, R_{eq} = 38.3 Ω, X_{eq} = 192 Ω.

To convert the transformer to per-unit system, the primary circuit base impedance needs to be found.

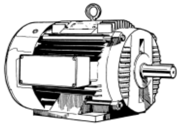
$$V_{base1} = 8\,000\,V; \quad S_{base1} = 20\,000\,VA$$

$$Z_{base1} = \frac{V_{base1}^2}{S_{base1}} = \frac{8\,000^2}{20\,000} = 3\,200\,\Omega$$

$$Z_{SE,pu} = \frac{38.4 + j192}{3\,200} = 0.012 + j0.06\ pu$$

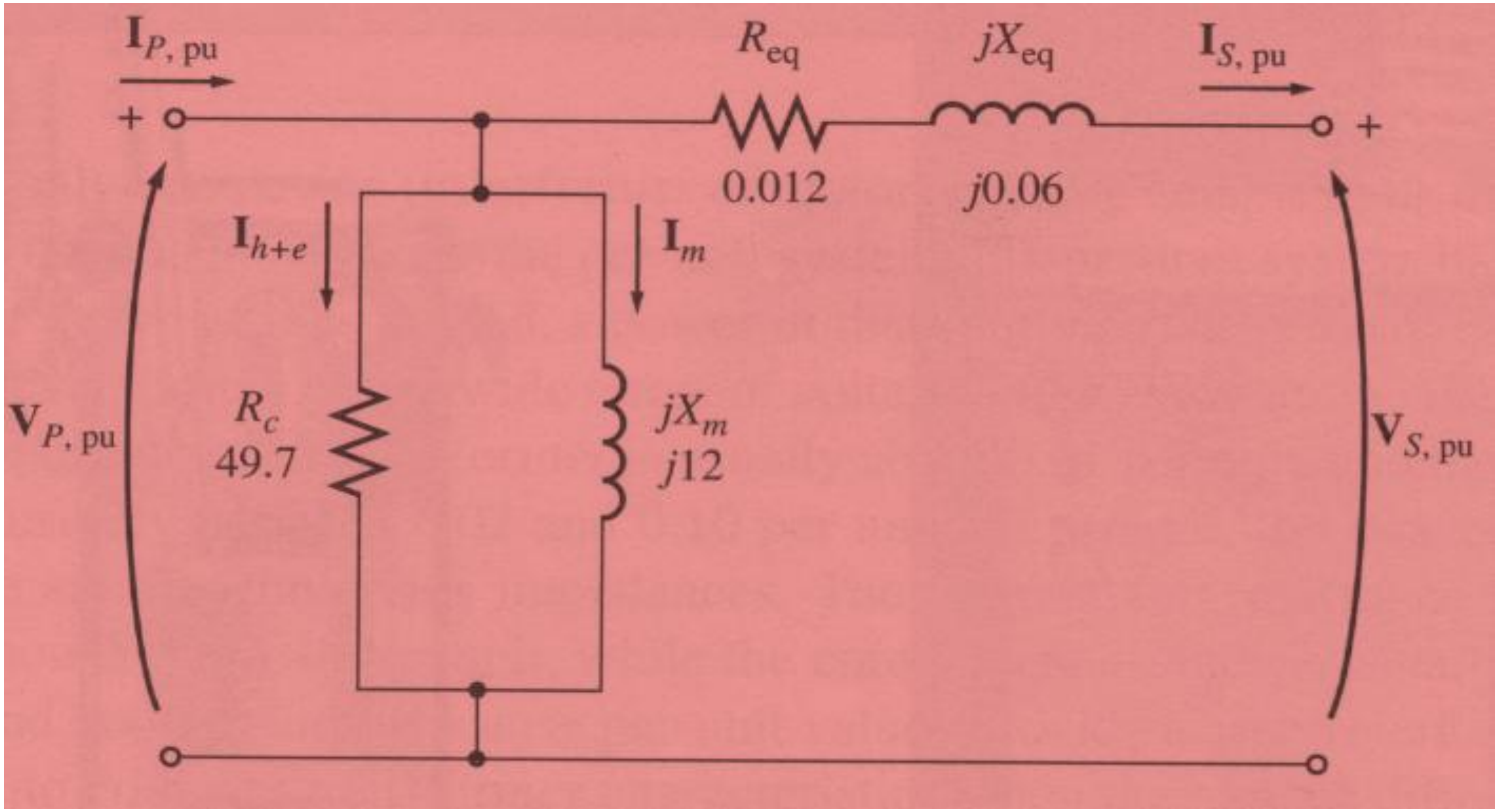
$$R_{C,pu} = \frac{159\,000}{3\,200} = 49.7\ pu$$

$$X_{M,pu} = \frac{38\,400}{3\,200} = 12\ pu$$



The per-unit system: Example

Therefore, the per-unit equivalent circuit is shown below:


$\mathbf{I}_{P,\,\mathrm{pu}}$
R_{eq}
jX_{eq}
$\mathbf{I}_{S,\,\mathrm{pu}}$
+
+
0.012
j0.06
$\mathbf{I}_{h+e}$
$\mathbf{I}_m$
$\mathbf{V}_{P,\,\mathrm{pu}}$
R_c
49.7
jX_m
j12
$\mathbf{V}_{S,\,\mathrm{pu}}$

Voltage regulation and efficiency

Since a real transformer contains series impedances, the transformer's output voltage varies with the load even if the input voltage is constant. To compare transformers in this respect, the quantity called a full-load voltage regulation (VR) is defined as follows:

$$VR = \frac{V_{s,nl} - V_{s,fl}}{V_{s,fl}} \cdot 100\% = \frac{V_p / a - V_{s,fl}}{V_{s,fl}} \cdot 100\% \qquad (4.52.1)$$

In a per-unit system:

$$\boxed{VR = \frac{V_{p,pu} - V_{s,fl,pu}}{V_{s,fl,pu}} \cdot 100\%} \qquad (4.52.2)$$

Where $V_{s,nl}$ and $V_{s,fl}$ are the secondary no load and full load voltages.